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WORKSHOP ENGINEERING CALCULATIONS AND TECHNICAL SCIENCE

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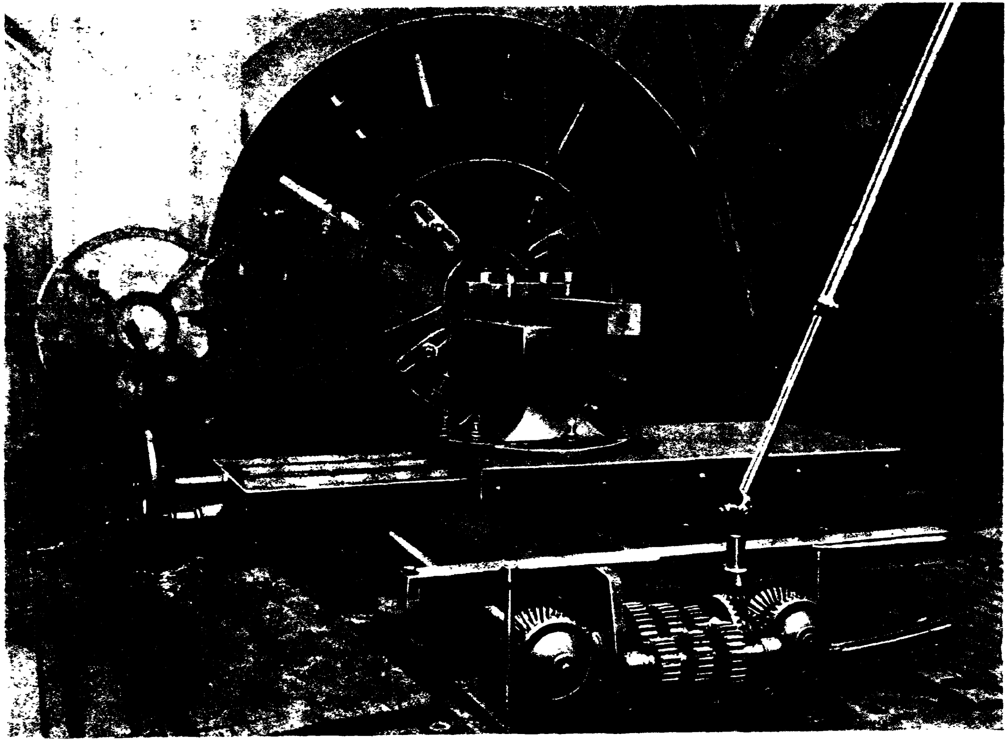

THE LATHE WHICH WAS USED BY JAMES WATT, THE GREAT ENGINEER AND INVENTOR. IT MAY BE SEEN AT THE WORKS OF W. & T. AVERY, SMETHWICK, BIRMINGHAM; WHERE MANY OF WATT'S PRODUCTIONS WERE MADE.

[*Frontispiece*

THE TECHNICAL COLLEGE SERIES

WORKSHOP ENGINEERING CALCULATIONS AND TECHNICAL SCIENCE

By

J. STONEY, B.Sc.
(Engineering), London.
WOLVERHAMPTON AND STAFFORDSHIRE
TECHNICAL COLLEGE

FIRST YEAR COURSE

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GENERAL EDITOR'S FOREWORD

TECHNICAL education stands on the verge of a great advance, probably the greatest in its history. Throughout the country there is general recognition of the necessity for a great expansion of the facilities which are available for thorough and systematic training for science and industry. This desire finds expression in the provision which has been made in the new Education Act for all forms of Technical Education. During the war many thousands of men and women have been compelled to undergo brief and hastily improvised courses in scientific and technical processes, and this has helped to bring home to them, and to most people, the importance of systematic technical education in the conditions of modern life.

There is general recognition of the well-founded conclusions of experts that the economic prosperity of this country will, more than ever before, be dependent upon the efficiency and adaptability of our system of scientific and technical training.

It is the hope and ambition of those who have been planning this series of technical books that it will meet the demands for new books which will follow these developments. New measures require new books, and it is hoped that this new series will not only assist in meeting these requirements but will incorporate those new methods and processes which have been introduced during the war years.

Fundamentally the books of the series are based, whenever this is necessary, on the Course system, which is now an established feature of technical education. Accordingly, in basic subjects there will be suitable volumes for the successive years of the course. It is recognised that there must be differences of opinion among teachers as to the sequence and contents of the work in the different years of a Course. But in general it is hoped that the treatment of the subjects will be sufficiently flexible to meet the needs of most teachers and students.

P. ABBOTT.

INTRODUCTION

THE change taking place in teaching the elementary grades of engineering subjects in technical institutions at the present time makes it necessary to co-ordinate text-books on the subject with the newer methods of teaching. Whereas, only a few years ago, emphasis was laid on teaching a pupil to understand the scientific and mathematical processes involved when a steam engine, internal-combustion engine, or other generators and machines are at work, equal importance is now laid on having a trained concept of the processes involved in the manufacture of the above-mentioned appliances or other machines and objects.

This text-book has been written with the object of co-operating with the newer method of teaching engineering subjects, and to harmonise with the revised suggestions and syllabuses of such public examining institutions as The City Guilds of London, the Lancashire and Cheshire Institutes, the Midland Counties Institutes, etc.

Thanks are due to several engineering firms for the privilege of allowing photographs of machinery and other material to be inserted in the book : Messrs. W. and T. Avery, Ltd., Messrs. Crofts, Ltd., Messrs. The Cementation Co., Ltd., Messrs. The English Steel Corporation, Messrs. J. Hopkinson & Co., and Messrs. Alfred Herbert, Ltd.

Exercises have been added at the end of each chapter, and answers to them all will be found at the end of the book. It is a great advantage for a technical student to work a large number of exercises in order to broaden his outlook on a subject, and a special feature has been made of this necessity by adding a fairly numerous quantity of them at the end of each section of the work dealt with in the book. Answers have been added in order that the student can check his work and the accuracy of his calculations.

The present volume deals with the work which may be accomplished in the first year of study in the subject.

The FIRST YEAR work deals with introducing a knowledge of WORKSHOP MATHEMATICS, MECHANICS, and HEAT: Extensions are made of all three subjects in Book 2 ; making it suitable for students preparing for the second and third year's examinations in GENERAL WORKSHOP ENGINEERING and similar tests.

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CHAPTER I

SCALE FRACTIONS

IN any ordinary fraction, such as $\frac{7}{16}$, the number **above the line** is called the **numerator,** while the number **below** is called the **denominator.** In scale fractions it will be observed that the denominators are 2 or multiples of 2. For example, in the scale fractions:

$$\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{7}{16}, \frac{21}{32},$$

each denominator is twice the one preceding it. Thus there is an easy rule for adding scale fractions, as shown below: viz. change all the denominators so that each is equal to the biggest of them. To do so, make use of the following simple rule: *If both the numerator and denominator of any fraction be multiplied by the same number whatsoever, the value of the fraction is not altered.*

Example (1). Take the fraction $\frac{1}{2}$; multiply numerator and denominator by 4. The result is $\frac{4}{8}$, which of course is equal to one-half. Two lines used thus = mean " equal to."

Example (2). $\frac{1 \times 16}{2 \times 16} = \frac{16}{32}$, which is equal to one-half.

Plus and Minus Signs

The sign + when used in arithmetic is called "**plus**" and means "**add**" or " added to."

The sign — is called "**minus**" and means "**take away**" or "subtract."

Mixed Numbers

Numbers like $3\frac{5}{8}$ and $1\frac{5}{16}$, which are partly whole number and partly fraction, are called **mixed numbers.**

To change a mixed number wholly into a fraction proceed as in the following examples:

Example (1). $$3\frac{5}{8} = \frac{(3 \times 8) + 5}{8}$$
$$= \frac{24 + 5}{8} = \frac{29}{8}.$$

Example (2). $$1\frac{5}{16} = \frac{(1 \times 16) + 5}{16} = \frac{16 + 5}{16}$$
$$= \frac{21}{16}.$$

To change a fraction in which the numerator is greater than the denominator into a mixed number the above process is reversed.

Example (1). $\frac{21}{16} = \frac{(1 \times 16) + 5}{16} = \frac{16}{16} + \frac{5}{16} = 1\frac{5}{16}$.

Example (2). $\frac{29}{8} = \frac{(3 \times 8) + 5}{8} = 3 + \frac{5}{8} = 3\frac{5}{8}$.

Addition and Subtraction of Fractions

Rule.—Change the fractions so that they all have the same denominators, as shown above, then add or subtract the numerators.

Example (1). *Add the fractions* $\frac{1}{2} + \frac{7}{8} + \frac{3}{32}$.

The greatest denominator is 32. This is called the *common denominator*. Make all the denominators into 32 by the rule stated on page 1.

$$\begin{aligned}\tfrac{1}{2} + \tfrac{7}{8} + \tfrac{3}{32} &= \tfrac{16}{32} + \tfrac{28}{32} + \tfrac{3}{32} \\ &= \frac{16 + 28 + 3}{32} \\ &= \tfrac{47}{32} \\ &= 1\tfrac{15}{32}.\end{aligned}$$

Example (2). *Find the value of* $\frac{3}{4} - \frac{7}{32}$.

Changing the denominator of the first fraction to 32.

$$\begin{aligned}\tfrac{3}{4} - \tfrac{7}{32} &= \frac{8 \times 3}{8 \times 4} - \tfrac{11}{32} \\ &= \tfrac{24}{32} - \tfrac{7}{32} \\ &= \tfrac{17}{32}.\end{aligned}$$

Example (3). *Find the sum of the fractions:*

$$\tfrac{15}{16} + \tfrac{7}{8} - \tfrac{27}{32} + \tfrac{3}{4} - \tfrac{1}{2} + \tfrac{13}{16} - \tfrac{11}{32}.$$

Changing all the denominators to 32, these become:

$$\tfrac{30}{32} + \tfrac{28}{32} - \tfrac{27}{32} + \tfrac{24}{32} - \tfrac{16}{32} + \tfrac{26}{32} - \tfrac{11}{32}.$$

The numerators with the minus sign in front must be subtracted, while the others must be added, thus:

$$\begin{aligned}30 + 28 + 24 + 26 - 27 - 16 - 11 &= 108 - 54 \\ &= 54.\end{aligned}$$

The result is $\frac{54}{32}$ or $1\frac{22}{32}$.

Factors. Cancelling of Factors.

A factor is a number which will divide into another number and leave no remainder. The figure 2 will divide the numerator and denominator of the above fraction, $\frac{22}{32}$, this is called *cancelling a common factor*. The result is $1\frac{11}{16}$. It is an advantage to reduce the numbers in a fraction as low as possible by cancelling.

Example (1). *Obtain the sum of the following :*

$$2\tfrac{3}{4} + 3\tfrac{7}{8} - 4\tfrac{11}{16}.$$

First sum the whole numbers :

$$2 + 3 - 4 = 1.$$

Now sum the fractions :

$$\tfrac{12}{16} + \tfrac{14}{16} - \tfrac{11}{16} = \tfrac{15}{16}.$$

The total $= 1\frac{15}{16}$.

Example (2). *Find the sum of the following dimensions :*

$$2\tfrac{5}{8}'' + 7\tfrac{3}{16}'' + 9\tfrac{3}{4}''.$$

The sum of the whole numbers or integers is 18. Adding the fractions : $\frac{10}{16} + \frac{3}{16} + \frac{12}{16} = \frac{25}{16}$.

Result is $18\frac{25}{16}$ or $19\frac{9}{16}''$.

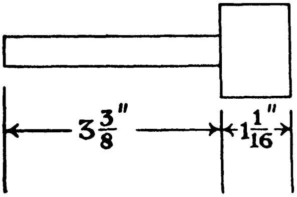


FIG. 1.

Example (3). *For the pin shown in Fig.* 1, *allow* $\frac{1}{16}''$ *at each end for facing and* $\frac{3}{16}''$ *for the width of the saw cut. Find the length to be marked on the bar from which the metal is cut.*

$$\text{Length} = 3\tfrac{3}{8} + 1\tfrac{1}{16} + \tfrac{1}{16} + \tfrac{1}{16} + \tfrac{3}{16}$$
$$= 4\tfrac{12}{16} = 4\tfrac{3}{4}''.$$

Example (4). *In Fig.* 2, *find the distance marked D.*

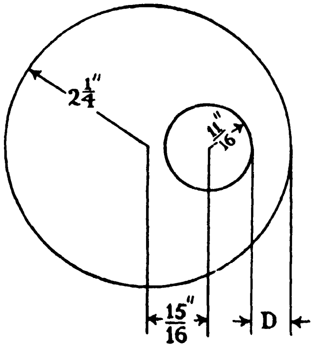


FIG. 2.

$$D = 2\tfrac{1}{4} - \tfrac{15}{16} - \tfrac{11}{16}$$
$$= 2\tfrac{1}{4} - \tfrac{26}{16}$$
$$= 2\tfrac{4}{16} - 1\tfrac{10}{16} = 1\tfrac{4}{16} - \tfrac{10}{16} = \tfrac{16}{16} + \tfrac{4}{16} - \tfrac{10}{16} = \tfrac{20}{16} - \tfrac{10}{16}$$
$$= \tfrac{10}{16}'' = \tfrac{5}{8}''.$$

Example (5). *In Fig.* 3, *find the complete length of the pin. The complete length of the pin is :*

$$3'' + \tfrac{5}{8}'' + \tfrac{1}{8}'' + \tfrac{15}{16}'' = 4\tfrac{11}{16}''.$$

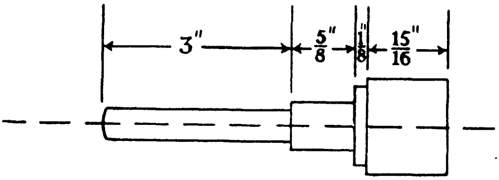


FIG. 3.

Multiplication of Fractions

(1) Multiplication of a fraction by a whole number.

Rule.—*Multiply the numerator of the fraction by the whole number.*

If it contains a whole number and a fraction, multiply the whole number separately and add the result of the multiplication of the fraction.

Example (1). $\tfrac{3}{8} \times 12 = \dfrac{3 \times 12}{8} = \tfrac{36}{8} = 4\tfrac{4}{8}$
$= 4\tfrac{1}{2}$.

Example (2). *What total length of bar is required for* 25 *pins such as shown in Fig.* 1 ?

Total length $= 4\tfrac{3}{4} \times 25$
$= (4 \times 25) + (\tfrac{3}{4} \times 25)$
$= 100 + \tfrac{75}{4} = 100 + 18\tfrac{3}{4}$
$= 118\tfrac{3}{4}$ inches.

(2) Multiplication of a fraction by a fraction.

Rule.—*Multiply the numerators for the new numerator, and multiply the denominators for the new denominator.*

Example (1). $\tfrac{5}{16} \times \tfrac{3}{4} = \dfrac{5 \times 3}{16 \times 4}$
$= \tfrac{15}{64}$.

Example (2). *Multiply* $1\tfrac{1}{2} \times 2\tfrac{3}{4}$.

In such examples express the numbers as fractions. Thus :

$1\tfrac{1}{2} = \tfrac{3}{2}$ and $2\tfrac{3}{4} = \tfrac{11}{4}$.

$\therefore 1\tfrac{1}{2} \times 2\tfrac{3}{4} = \tfrac{3}{2} \times \tfrac{11}{4}$
$= \dfrac{3 \times 11}{2 \times 4} = \tfrac{33}{8}$
$= 4\tfrac{1}{8}$.

Note.—The sign ∴ means " therefore."

Division of Fractions

Rule.—*Invert the fraction by which you are dividing and multiply the two fractions.*

Note. The sign ÷ means " divide by."

Example (1). $\frac{11}{16} \div \frac{5}{8}$.

$$\frac{11}{16} \div \frac{5}{8} = \frac{11}{\underset{2}{\cancel{16}}} \times \frac{\overset{1}{\cancel{8}}}{5}$$
$$= \frac{11}{10}$$
$$= 1\frac{1}{10}.$$

Example (2). $\frac{21}{32} \div 12 = \frac{21}{32} \times \frac{1}{12} = \frac{\overset{7}{\cancel{21}} \times 1}{32 \times \underset{4}{\cancel{12}}} = \frac{7}{128}$

Example (3). $3\frac{3}{8} \div \frac{7}{18} = \frac{27}{\underset{4}{\cancel{8}}} \times \frac{\overset{9}{\cancel{18}}}{7} = \frac{243}{28} = 8\frac{19}{28}$.

Example (4). *Divide* $3\frac{5}{8}$ by $1\frac{5}{16}$.

$$\text{e.g. } \frac{\frac{29}{8}}{\frac{21}{16}} = \frac{29}{\underset{1}{\cancel{8}}} \times \frac{\overset{2}{\cancel{16}}}{21} = \frac{58}{21}$$
$$= 2\frac{16}{21}.$$

Fractions with Other Denominators than 2 or multiples of 2.

Examples. $\frac{5}{9}$, $\frac{13}{17}$, etc.

In general, all fractions which contain numerator and denominator, whether they are **scale** fractions or *not*, are called **vulgar** fractions. This title serves to distinguish them from DECIMAL fractions, which are considered in Chapter II.

Summation

Example (1). *Find the sum of the fractions :*

$$\frac{2}{3} + \frac{7}{12} + \frac{2}{15}.$$

First, the LOWEST COMMON DENOMINATOR must be found.

Place the denominators thus : 3 12 15

Divide by a factor that will go into them all if possible :

$$\begin{array}{r|rrr} 3 & 3 & 12 & 15 \\ \hline & 1 & 4 & 5 \end{array}$$

No other factor will go into two of the numbers 1, 4, 5. Then multiply all the figures left together, including the dividing factors, *e.g.*

$$5 \times 4 \times 1 \times 3 = 60.$$

Hence 60 is the lowest common denominator of the three fractions.

Change all the denominators in the three fractions to 60.

$$\frac{2}{3} = \frac{40}{60}. \quad \frac{7}{12} = \frac{35}{60}. \quad \frac{2}{15} = \frac{8}{60}.$$

Adding: $\frac{2}{3} + \frac{7}{12} + \frac{2}{15} = \frac{40}{60} + \frac{35}{60} + \frac{8}{60} = \frac{83}{60} = 1\frac{23}{60}$.

Example (2). *Find the sum of :*

$$\frac{2}{3} + \frac{3}{7} + \frac{5}{8} - \frac{13}{21} + \frac{5}{12} + \frac{3}{14}.$$

$$\begin{array}{r|rrrrrr} 3 & 3 & 7 & 8 & 21 & 12 & 14 \\ \hline 7 & 1 & 7 & 8 & 7 & 4 & 14 \\ \hline 2 & 1 & 1 & 8 & 1 & 4 & 2 \\ \hline 2 & 1 & 1 & 4 & 1 & 2 & 1 \\ \hline & 1 & 1 & 2 & 1 & 1 & 1 \end{array}$$

Again, find the lowest common denominator. In this case, no factor will divide into them all, so divide by a number that will go into most of them, *i.e.* 3 or 2.

Now repeat the process, as long as we can divide into two of them. Multiplying up, the lowest common denominator is $3 \times 7 \times 2 \times 2 \times 2$, *i.e.* = 168.

One may make all the denominators equal to 168 as shown in Example (1), or proceed thus :

$$\text{Sum} = \frac{112 + 72 + 105 - 104 + 70 + 36}{168}$$

with each fraction, divide 168 by the denominator and then multiply the result by the numerator. Thus with the fraction $\frac{2}{3}$, 3 into 168 is 56. Then $56 \times 2 = 112$. Place this on top of the line as shown.

Again with $\frac{3}{7}$: 7 into 168 = 24. $24 \times 3 = 72$. Proceed thus with all the fractions.

Now take the sum of the numbers above the line, i.e. 291.

$$\text{Then } \frac{291}{168} = 1\frac{123}{168} = 1\frac{41}{56} \text{ is the result.}$$

Example (3). *Find the value of :* $3\frac{1}{7} + 2\frac{5}{21} - 2\frac{1}{12}$.

First sum the whole numbers : $3 + 2 - 2 = 3$.
Now sum the fractions.
The lowest common denominator is 84.

$$\frac{12 + 20 - 7}{84} = \frac{25}{84}.$$

The answer is $3 + \frac{25}{84} = 3\frac{25}{84}$.

EXERCISE 1

(1) $\frac{3}{8} \times \frac{1}{4}$. (2) $3\frac{7}{12} \times 3$. (3) $12\frac{12}{13} \times \frac{1}{4}$.
(4) $\frac{3}{4} + \frac{7}{8}$. (5) $\frac{5}{16} + \frac{11}{64}$. (6) $\frac{11}{32} - \frac{1}{4}$.
(7) $\frac{7}{8} - \frac{3}{16} + \frac{3}{32}$. (8) $\frac{31}{64} + \frac{11}{32} - \frac{5}{8}$. (9) $\frac{3}{4} \times \frac{5}{16}$.
(10) $1\frac{1}{4} \times \frac{4}{5}$. (11) $\frac{7}{8} \div \frac{3}{16}$. (12) $\frac{3}{8} \div \frac{11}{16}$.
(13) $\frac{15}{16} \div 3$. (14) $\frac{21}{64} \div 7$. (15) $3\frac{1}{8} \div 1\frac{1}{4}$.
(16) $1\frac{9}{64} \div \frac{5}{16}$. (17) $\frac{3}{4} \times \frac{1}{8}$. (18) $2\frac{1}{2} \times 1\frac{1}{4}$.
(19) $\frac{15}{16} \div \frac{1}{4}$. (20) $\frac{7}{8} + \frac{1}{4} - \frac{7}{8} \times \frac{1}{4}$.

(21) Twelve lengths of metal rod, $7\frac{1}{8}''$ long, are required to be cut from a bar. Allow $\frac{3}{64}''$ at each cut for the width of the saw, and calculate the total length cut from the bar.

(22) Three lots of scrap bronze weigh 12 lb. 7 oz., 5 lb. 13 oz., and 8 lb. 5 oz., respectively. Find the total weight. (Note, one ounce is $\frac{1}{16}$th of a lb.)

(23) In the pin bolt shown in Fig. 4, allow $\frac{1}{16}''$ at each end for finishing and $\frac{1}{8}''$ for the width of the saw-cut, and calculate the length of bar for twelve such pin bolts.

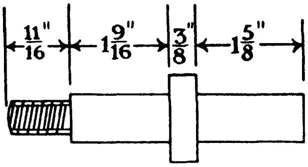


FIG. 4.

(24) For the slotted plate shown in Fig. 5, allow $\frac{3}{16}''$ along each edge for filing and finish, and calculate the size of plate to be cut from the sheet.

(25) Find the sum of $\frac{3}{4} + \frac{7}{12} - \frac{11}{24}$.

(26) Multiply $2\frac{1}{3}$ by 10, by 5, and by 4.

(27) Find the value of $6\frac{22}{57}$ multiplied by $2\frac{12}{13}$.

(28) Evaluate $\frac{2}{3}$ of $\frac{1}{9} \times \frac{7}{11}$.

(Note : " of " in this connection is equivalent to " × .")

(29) Divide $\frac{12}{27}$ by $\frac{9}{13}$.

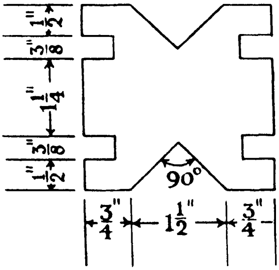


FIG. 5.

(30) Evaluate $9\frac{3}{8} \div 8\frac{7}{16}$.

(31) Twenty-five lengths of steel rod, $3\frac{1}{2}''$ long, are required to be sawn from a long bar. Allow $\frac{1}{12}''$ for the width of each saw-cut, and calculate the total length taken from the rod.

(32) If the length of the bolt shown in Fig. 4 above had to be reduced to $\frac{3}{4}$ the length, what would be its new length ?

CHAPTER II

DECIMAL FRACTIONS

WHEREAS ordinary scale fractions have denominators which are either 2 or multiples of 2, decimal fractions have denominators which are either 10 or multiples of 10. But instead of writing the fraction, say $\frac{3}{10}$, as an ordinary fraction, an easy conventional method has been adopted: thus $\frac{3}{10}$ is written ·3. The dot before the number is called the decimal point, and the first figure following shows the number of tenths. The second figure following the decimal point indicates the number of hundredths. Thus $\cdot 34 = \frac{3}{10} + \frac{4}{100}$. The third figure following the decimal point shows the number of thousandths, and the fourth figure the ten-thousandths, and so on. A whole number may be placed before the decimal point, *e.g.* 2·3479. Thus—

$$2\cdot 3479 = 2 + \frac{3}{10} + \frac{4}{100} + \frac{7}{1000} + \frac{9}{10000}.$$

Addition

17·32 + ·891 + ·0079.

Arrange the numbers in columns so that the decimal points are under one another. Thus—

$$\begin{array}{rr} & 17\cdot 32 \\ & \cdot 891 \\ & \cdot 0079 \\ \hline \text{Ans.} & 18\cdot 2189 \\ \hline \end{array} \quad \textit{Add up the columns.}$$

Subtraction

Take 2·7913 from 12·5.

Place the decimal points in column.

$$\begin{array}{r} 12\cdot 5 \\ 2\cdot 7913 \\ \hline \end{array}$$

There appear to be no figures from which to subtract the figures 913. In such a case imagine 0's following the 5: then proceed as in ordinary subtraction.

$$\begin{array}{r} 12\cdot 5000 \\ 2\cdot 7913 \\ \hline 9\cdot 7087 \\ \hline \end{array}$$

Multiplication

2·34 × 7·6.

Proceed as in ordinary multiplication.

$$\begin{array}{r} 2{\cdot}34 \\ \underline{7{\cdot}6} \\ 1404 \\ \underline{1638} \\ \underline{17{\cdot}784} \end{array}$$

To place the decimal point in the answer—

Count the number of decimal figures in the two numbers. In this case there are three. Count the same number of figures from the right-hand end of the answer and then place the decimal point in front of them.

To multiply a decimal fraction by a multiple of 10, *move the decimal point one place to the right for each* " 0 " *the multiplier contains.* Thus :

$$\begin{array}{rcl} 2{\cdot}732 \times 100 &=& 273{\cdot}2. \\ {\cdot}07932 \times 10000 &=& 793{\cdot}2. \\ {\cdot}4329 \times 10 &=& 4{\cdot}329. \end{array}$$

Division

Example. *Divide* 36·425 *by* 3·12.

This may be written in the form $\frac{36{\cdot}425}{3{\cdot}12}$

In dividing by a decimal fraction it tends to greater accuracy in results if the decimal point is removed from the denominator. To do this one multiplies both numerator and denominator by such a number as will convert the denominator into a whole number. Thus in the above example multiply by 100.

$$\frac{3642{\cdot}5}{312}$$

Then begin as in ordinary long division.

$$\begin{array}{rl} 312)3642{\cdot}5(11{\cdot}6746 & \\ \underline{312\phantom{0{\cdot}5}} & \\ 522\phantom{{\cdot}5} & \\ \underline{312\phantom{{\cdot}5}} & \\ \leftarrow 2105\phantom{{\cdot}} & \\ \underline{1872\phantom{{\cdot}}} & \\ 2330 & \leftarrow \text{As there are no further figures add 0.} \\ \underline{2184} & \\ 1460 & \\ \underline{1248} & \\ 2120 & \\ \underline{1872} & \\ \underline{248} & \end{array}$$

The next figure to → be brought down is the ·5. Before doing so, put the decimal point in the answer.

Give the answer correct to the third decimal place, that is to the nearest $\frac{1}{1000}$th.

Rule.—When the fourth figure is 5 or more than 5, add one to the third figure and cross out all following figures.

Answer = 11·675.

To divide a number by 10 or a power of 10.

Move the decimal point one place to the left for each "0" the divisor contains. Thus:

$$72·056 \div 100 = ·72056.$$
$$2·976 \div 1000 = ·002976$$
$$5762 \div 10000 = ·5762.$$

To convert a vulgar fraction into a decimal fraction.

Example. Convert $\frac{11}{16}$ to a decimal fraction.

16)11 (

Sixteen won't go into 11, so place the decimal point in the answer and then add a "0" to the 11. Then divide thus:

16)110(·6875
96
140
128
120
112
80
80
. .

Thus $\frac{11}{16}$ is equal to ·6875.

Example. *Convert $\frac{3}{7}$ to a decimal fraction.*

7)3·0000
·42857

This fraction will produce a long string of decimals, which in fact will never end. Consequently the division is continued as far as is necessary to give any required degree of accuracy.

To convert a decimal fraction into a vulgar fraction.

Example (1). *Convert ·762 to a vulgar fraction.*

Rule.—In the denominator place a "1" and as many 0's as there are decimal figures.

$$\therefore ·762 = \frac{762}{1000}$$

Example (2). $·007103 = \frac{7103}{1000000}.$

Of course, after setting out the fractions as above, cancelling in numerator and denominator may be done if possible.

Decimal fraction involving weights and measures.

Example (1). *Find the value of* 0·625 *of a ton, stating the answer in lb.*

0·625 of 2240.

As in the cases of vulgar fractions, the word " of " may be replaced by the multiplication sign.

Multiplying here we get :

$$0{\cdot}625 \times 2240 = 1400 \text{ lb.}$$

Example (2). *What decimal fraction of a full day is* 9 *hours ?*

$$\tfrac{9}{24} = \tfrac{3}{8} = {\cdot}375.$$

Example (3). 0·35 *of a certain brass is zinc. Find the weight of zinc in* 2400 *lb. of this brass.*

$$0{\cdot}35 \times 2400 = 840 \text{ lb.}$$

Table of Decimal Equivalents to Scale Fractions

EIGHTHS	SIXTEENTHS
$\frac{1}{8}$ = ·125	$\frac{1}{16}$ = ·0625
$\frac{2}{8}$ = ·250	$\frac{3}{16}$ = ·1875
$\frac{3}{8}$ = ·375	$\frac{5}{16}$ = ·3125
$\frac{4}{8}$ = ·500	$\frac{7}{16}$ = ·4375
$\frac{5}{8}$ = ·625	$\frac{9}{16}$ = ·5625
$\frac{6}{8}$ = ·75	$\frac{11}{16}$ = ·6875
$\frac{7}{8}$ = ·875	$\frac{13}{16}$ = ·8125
	$\frac{15}{16}$ = ·9375

BRITISH STANDARD, WHITWORTH, THREADS

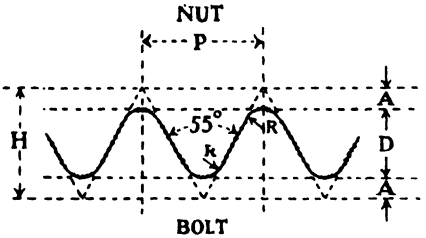


FIG. 6.

$$P = \text{Pitch} = \frac{1}{\text{No. of threads per inch}}.$$

$$H = \text{Theoretical depth} = {\cdot}9605 \times P.$$

$$D = \text{Actual depth} = {\cdot}6403 \times P.$$

Rounding at Crest and Root $= A = \frac{H}{6} = \cdot 1601 \times P.$

Radius at Crest and Root $= R = \cdot 1373 \times P.$

Example. One-inch bolt, 8 threads to the inch length of bolt.

$$\begin{aligned} \text{Pitch} &= \tfrac{1}{8}'' = \cdot 125'' \\ H &= \tfrac{1}{8} \times \cdot 9605 = \cdot 1201'' \\ D &= \cdot 6403 \times \tfrac{1}{8} = \cdot 080'' \\ A &= \cdot 1601 \times \tfrac{1}{8} = \cdot 020'' \\ R &= \cdot 1373 \times \tfrac{1}{8} = \cdot 017'' \end{aligned}$$

Tolerance

It is customary to state the error one would **tolerate** when work is demanded to be of a very considerable accuracy. For example, a turner may be given instructions on the blue print to turn a metal cylinder to a diameter $4{\cdot}5'' \pm \cdot 0005''$. The sign ± means plus or minus, so that the result is to be between $4{\cdot}5'' + \cdot 0005$ and $4{\cdot}5'' - \cdot 0005$. This would mean that the work would be passed as satisfactory if it were not more than five ten-thousandths of an inch above or below 4·5 inches. The highest value the diameter could have would be 4·5005″, and the lowest value the diameter could have would be 4·4995″: these numbers are called the **upper and lower limits** respectively.

The difference between the two limits is called the **TOLERANCE.** In the above case the tolerance is 0·001″, i.e. one-thousandth of an inch.

Example

$$1{\cdot}75'' \begin{matrix} + \cdot 004'' \\ - \cdot 002'' \end{matrix}$$

In this case the upper limit is 1·754″ and the lower limit is 1·748″.

The **tolerance is 0·006″.**

If a large number of the same thing is required, then a special gauge of the upper and lower limits is usually given to the operative for use.

The following figures illustrate types of gauges: the first gauge is a type which is used for measuring the diameter of a shaft or other outside measurement.

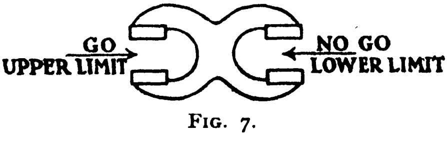


FIG. 7.

For measuring the internal diameters the following is a type of gauge commonly used. They are named GO and NO GO gauges. One end is the upper limit of the diameter and must not go in; the other is the lower limit and must go in.

Plug Gauge.
For internal dimensions.

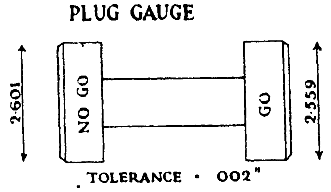


Fig. 8.

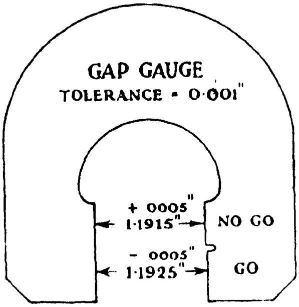


Fig. 9.

Gap Gauge for outside measurements

Depth Gauge reading $2\frac{1}{16}''$.
Used for gauging the depths of cylinders, etc.

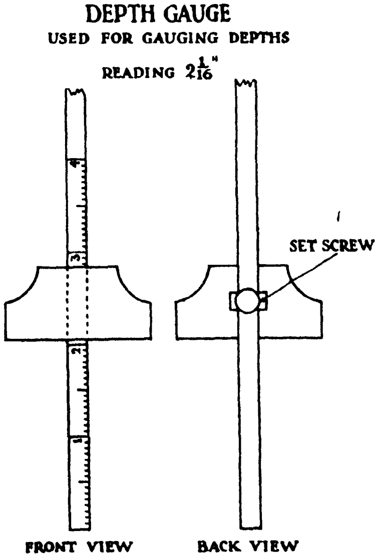


Fig. 10.

Feeler Gauges, to test joints, etc.

The numbers on the blades indicate the number of thousandths of an inch thick.

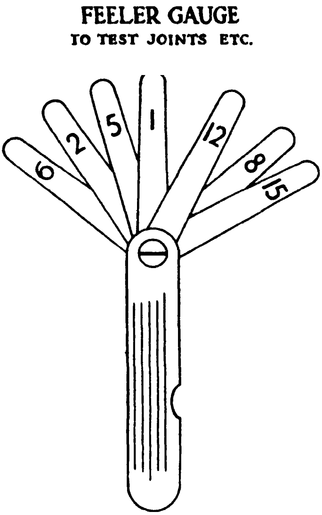


Fig. 11.

Micrometer Calipers

The following figure illustrates a micrometer caliper used for measuring the dimension of an object of size up to one inch, and will read the dimension to the nearest thousandth of an inch. It is set to read ·648″.

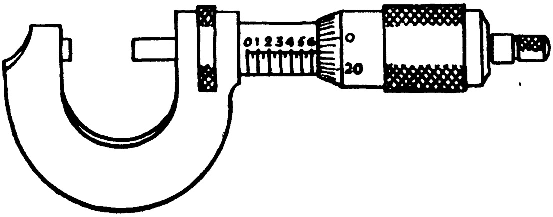


Fig. 12.

The scale shows tenths of an inch marked by the numbers 0, 1, 2 . . . 6 : each tenth is divided into four equal parts, i.e. fortieths of an inch.

Thus on the scale tenths and fortieths of an inch are read off, each $\frac{1}{40}$ being ·025″.

In the figure above six-tenths and one-fortieth are shown. Besides, the twenty-third division on the thimble is opposite the mark and this is an extra twenty-three thousandths.

Hence the reading is six-tenths, one-fortieth, and twenty-three thousandths, *i.e.*

$$\begin{array}{r} \cdot 6 \\ \cdot 025 \\ \cdot 023 \\ \hline \cdot 648'' \end{array}$$

The student easily gets into the habit of mentally computing the dimensions on the micrometer. The thimble has 25 equal divisions on it, each equal to ·001″. One complete revolution of the thimble advances the spindle $\frac{1}{40}''$ or ·025″.

The above remarks relate to the STANDARD MICROMETER. There are, however, other constructions of micrometers—some of them on the centimetre scale, reading to the thousandth of a centimetre.

The following figure shows an enlarged view of the above reading.

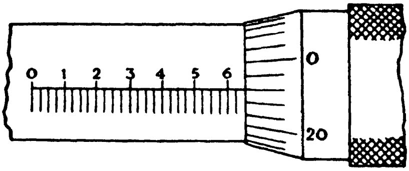


FIG. 13.

Other views and readings of micrometers follow with instructions on the readings. One of them shows a micrometer to read to the thousandth of a centimetre.

MICROMETER READING $\frac{1}{10}$th, $\frac{1}{50}$th & $\frac{1}{1000}$ inch

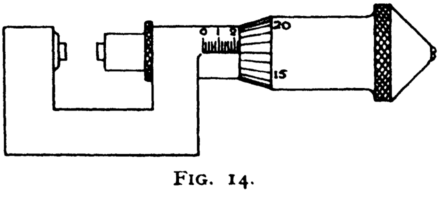


FIG. 14.

This micrometer has a scale of tenths of an inch. Each tenth is divided into five equal parts giving fiftieths. The thimble is divided into twenty equal parts, *i.e.* thousandths.

One complete revolution of the thimble means an advance of one-fiftieth of an inch. One division on the thimble $= \frac{1}{20} \times \frac{1}{50} = \frac{1}{1000}''$.

The reading is 0·2 + ·020 + ·017, i.e. = 0·237.

Micrometer reading to the thousandth of a centimetre.

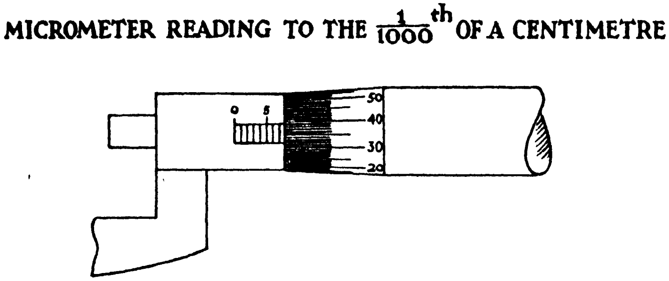

MICROMETER READING TO THE $\frac{1}{1000}$th OF A CENTIMETRE
0
5
50
40
30
20

Fig. 15.

The scale is divided into tenths or millimetres, and the thimble into a hundred equal parts.

One complete revolution of the thimble gives an advance of one-tenth centimetre, or one millimetre. Each mark in the thimble means 0·001 cm.

The reading is 0·739 cm.

Vernier Calipers.

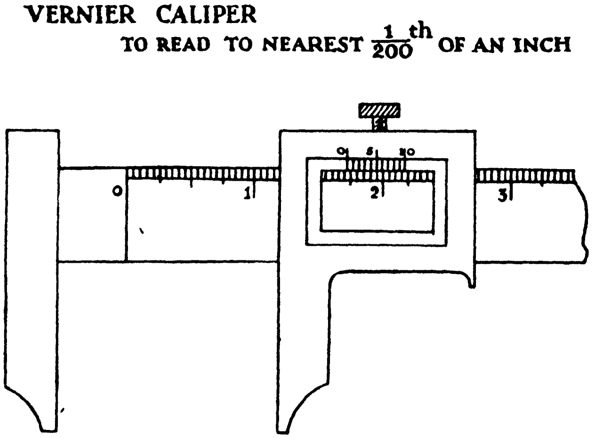


Fig. 16.

This vernier is graduated to read to one-two-hundredth of an inch. Ten divisions on the vernier cover nine on the scale. The inch is divided into twentieths : $\frac{1}{20}'' = \cdot050''$.

The reading is 1" + 14 twentieths + 5 two-hundredths.

$$1 + \cdot7 + \cdot025 = 1\cdot725''.$$

VERNIER TO READ THOUSANDTHS OF AN INCH. SET AT 1·456″.

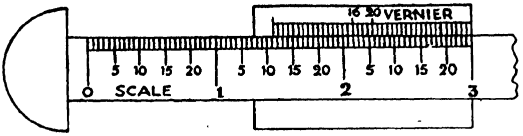


FIG. 17.

The inch on the scale is divided into twenty-five equal parts. Forty on the vernier cover thirty-nine parts on the scale. Each division on the vernier is one-fortieth of $\frac{1}{25}''$, or ·001″ less than a division on the scale.

Each $\frac{1}{25}'' = \cdot 040$. Thus the reading is one inch plus 11 twenty-fifths, plus 16 thousandths, *i.e* $1'' + 11 \times \cdot 040 + \cdot 016$.

This gives 1·456″.

VERNIER CALIPER TO READ TO NEAREST $\frac{1}{100}$ CM. VERNIER SET TO READ 3·33 CM.

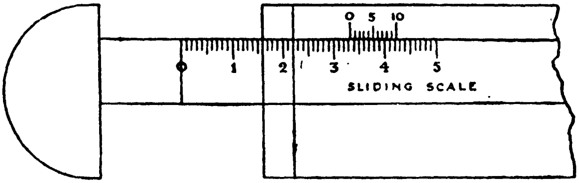


FIG. 18.

Ten divisions on vernier equal nine on sliding scale.

The distance between the jaws of the caliper is the same as the distance between the zero marks.

On the scale, one reads 3·3 cm. On the vernier the third mark is opposite.

Hence the reading is 3·33 cm.

EXERCISE 2

(1) Multiply the following decimal fractions by 1000 :

(*a*) ·7203. (*b*) ·00273. (*c*) ·2. (*d*) ·79.

(2) Divide the following numbers by 1000, leaving the results as decimal fractions :

(*a*) 2739·4. (*b*) 794·7. (*c*) 5·65. (*d*) ·3472.

(3) (*a*) $9 \cdot 375 \times 2 \cdot 4$. (*b*) $\cdot 0158 \times 6 \cdot 25$. (*c*) $2 \cdot 24 \times \cdot 75$.
(*d*) $7 \cdot 8125 \times 9 \cdot 6$. (*e*) $\cdot 3125 \times \cdot 24$. (*f*) $781 \cdot 25 \times 960$.

(4) (*a*) $15 \div \cdot 8$. (*b*) $\cdot 09 \div \cdot 4$. (*c*) $30 \div 2 \cdot 4$.

(5) (*a*) $3 \div \cdot 0625$. (*b*) $\cdot 455 \div 2 \cdot 6$. (*c*) $58 \cdot 77 \div 36$.

(6) The following table gives the full diameters and depths of thread for Whitworth screw threads.

Subtract twice the depth of thread from the full diameter and the result is the core diameter.

In the table fill in the core diameters.

Full diameter, inch	·25	·75	·5	1	1·5
Depth of thread, inch	·032	·064	·0534	·08	·1067
Core diameter, inch					

(7) The pitches of certain screws are :

(*a*) ·0625″. (*b*) ·125″. (*c*) ·1667″. (*d*) ·25″.

How many threads per inch length of screw are there ?

(8) The following dimensions are taken from a machine drawing :

(*a*) 36·4 mills. (*b*) 23·45 cm. (*c*) 36·5 cm.

Convert these dimensions to inches, given that there are 25·4 mills = 1″, also 2·54 cm. = 1″.

(9) A cylinder is 4·5″ bore and 7·4″ long, convert these dimensions to millimetres.

(10) In the table, fill in the vacant places :

Dimension required	2·5 ± ·003	4·75 ± ·002	1·256 ± ·0005
Upper limit	2·503		
Lower limit	2·497		
Tolerance	·006		

Simplify the fractions, numbered (11) to (14), and express the result as a decimal fraction :

(11) $\dfrac{5{\cdot}46 - 3}{3\frac{1}{2}}$. (12) $\dfrac{2{\cdot}46 - \frac{2}{3}}{1{\cdot}18}$.

(13) $\dfrac{2{\cdot}142}{72} \times \dfrac{150 \times 708}{56}$. (14) $\dfrac{22}{7}\left\{\dfrac{(3\frac{1}{2})^2 - (3\frac{1}{4})^2}{144}\right\}$.

(15) If the pitch of a screw is ·0625″, how many threads to the inch length are there ?

(16) A cubic inch of aluminium weighs ·092 lb. Find the weight of a block of aluminium whose volume is 38·5 cubic inches.

(17) Reduce 7·35 tons to pounds.

(18) Express 1107 yards as the decimal fraction of a mile. Answer to the third decimal place.

(19) Reduce 284·8 chains to miles.

(20) A man was offered in wages £5⅜ a week, or 17*s*. 6*d*. a day, or 2*s*. 2⅞*d*. an hour. The working week consists of 6 days and the working day of 8 hours. Which is the highest wage ?

(21) What decimal fraction of a ton is 1960 lb. ?

CHAPTER III

AVERAGES

THE average of two quantities is found by adding the two quantities together and dividing by two. It may be said to be the middle value between the two.

Example (1). *The lengths of two rods are* 13′ *and* 25′, *find the average of the two lengths.*

$$\text{Thus the average} = \frac{13 + 25}{2} = 19 \text{ feet.}$$

If the two rods were laid in a line end to end, it would be 19′ from one end to the middle.

Example (2). *The productions of a particular article on a certain day by two machines were* 371 *and* 353 *articles respectively. What was the average production by the two machines?*

$$\text{Average} = \frac{353 + 371}{2} = \frac{724}{2} = 362.$$

The average of a number of unequal quantities is found by adding all the numbers together and then dividing by the number of them. It may be called the "**Mean Amount**" or "**Mean Value**" of several unequal quantities or values.

Example (3). *Find the average of the following lengths:*

$$3'', 4\tfrac{1}{2}'', 3\tfrac{3}{4}'', 3\tfrac{1}{4}''.$$

$$\text{Average length} = \frac{3 + 4\frac{1}{2} + 3\frac{3}{4} + 3\frac{1}{4}}{4} = \frac{14\frac{1}{2}}{4}.$$

$$= 3\tfrac{5}{8}''.$$

To measure the diameters of wires, round rods, etc.

Several readings at different points on the length should be read and the average of them taken as being the correct diameter.

Example (4). *The following readings were made by a micrometer caliper in measuring the diameter of a rod. They were taken at different points in the length of the rod.*

2·32 cm., 2·324 cm., 2·319 cm., 2·321 cm.

To find the average of these readings, add the four numbers together and divide by four :

$$\begin{array}{r} 2{\cdot}32 \\ 2{\cdot}324 \\ 2{\cdot}319 \\ 2{\cdot}321 \\ \hline 4)9{\cdot}284 \\ \hline 2{\cdot}321 \\ \hline \end{array}$$

Hence **2·321** cm. may be taken as a correct record of the diameter of the rod.

Example (5). *The weekly wages of three men are* £4 17*s.* 6*d.*, £3 14*s.*, *and* £5 1*s.* 9*d. What is the average amount earned ?*

$$\begin{array}{rrr} \pounds & s. & d. \\ 4 & 17 & 6 \\ 3 & 14 & 0 \\ 5 & 1 & 9 \\ \hline 3)13 & 13 & 3 \\ \hline 4 & 11 & 1 \\ \hline \end{array}$$

£4 11*s.* 1*d.* is the average weekly wage.

Example (6). *The weights of three bags of bolts are* 2 *cwt.* 21 *lb.*, 5 *cwt.* 34 *lb.*, *and* 1 *cwt.* 36 *lb. What is the average weight of the bags ?*

$$\begin{array}{rr} \text{cwt.} & \text{lb.} \\ 2 & 21 \\ 5 & 34 \\ 1 & 36 \\ \hline 3)8 & 91 \\ \hline 2 & 105 \\ \hline \end{array}$$

Dividing 8 cwt. by 3 gives 2 cwt. over. Bring this to 224 lb.

$$\begin{array}{r} \therefore\ 224 \\ 91 \\ \hline 3)315 \\ \hline 105\ \text{lb.} \\ \hline \end{array}$$

∴ average weight of the bags is **2 cwt. 105 lb.**

Example (7). *In four different workshops the following consumptions of lubricating oil were registered during a certain period of working time :*

3¼ gallons, 4½ gallons, 2¾ gallons, and 3½ gallons.

What was the average consumption ?

$$\begin{array}{r} 3\tfrac{1}{4} \\ 4\tfrac{1}{2} \\ 2\tfrac{3}{4} \\ 3\tfrac{1}{2} \\ \hline 4)14 \\ \hline 3\tfrac{1}{2}\ \textbf{gallons.} \end{array}$$

EXERCISE 3

(1) Find the average of the following readings : 3·41″, 3·44″, 3·38″, 3·39″.

(2) The weekly amounts earned by three gangs of riveters are : £10 8*s*. 7½*d*., £10 8*s*. 6*d*., and £10 7*s*. 10½*d*. Find the average amount for the three gangs.

(3) Find the average weight of sixteen castings, four of which weigh 5⅛ stones each, and the rest weigh 6¾ stones each.

(4) Four bars of mild steel, each of them being ⅝″ diameter, were tested for their tensile strength. The breaking loads were 7½ tons, 8¼ tons, 8⅛ tons, and 7⅛ tons. What was the average breaking load ?

(5) Eight men working on a piecework job earned the following amounts during a particular day's labour :
18*s*. 7*d*., 18*s*. 4*d*., 18*s*. 11*d*., 18*s*. 5*d*., 18*s*. 7*d*., 17*s*. 7*d*., 18*s*. 4*d*., and 16*s*. 10*d*.

Find the average amount earned by the men.

(6) Eight deliveries of bolts were received in a stores. On weighing the eight quantities the amounts received were as follows—stated in stones and pounds :

8 3½, 9 4½, 11 3, 8 12½, 11 0½, 8 8½, 6 10¼, 8 6¾.

Find the average weight of the deliveries.

(7) A group of boys was engaged in a workshop and the following were their rates of pay :

4 boys got 3*s*. 4½*d*. each per day.
1 ,, ,, 2*s*. 6*d*. ,, ,, ,,
5 ,, ,, 2*s*. ,, ,, ,,
36 ,, ,, 3*s*. ,, ,, ,,

What is the average amount earned per day by the boys ? Give the answer to the nearest penny.

(8) In a cone pulley of three steps on a line shaft the following are the speeds of the belt in yards per minute. When on the largest step the speed is double that when on the middle step, and when on the middle step the speed is double that when on the smallest step. The average speed for the three cases is 154 yards a minute. Find the speed for each step.

(9) Four men working on a repetition job produced the following amounts in the times stated :

(1) 2466 articles in 197 days.
(2) 1878 ,, ,, 128 ,,
(3) 1095 ,, ,, 62 ,,
(4) 1153 ,, ,, 75 ,,

(*a*) Calculate the average number of articles per day produced by each man, giving the answers to the second decimal place.

(*b*) Also state the complete average per day for the four men taken as a whole.

(10) Find the average of the following numbers: 238, 218, 285, 321, 240, 183, 310, 184.

(11) In a heat-treatment operation, the following temperatures, Fahrenheit, of the oven were taken at regular intervals of time: 220°, 218°, 218°, 219°, 217°, 218°, 216°, and 220°. What is the average reading during the period?

(12) A planing-machine table when working makes the cutting stroke of 12 feet in 10 seconds and the return stroke in 8 seconds. What is the average speed in feet per minute?

(13) The average tonnage carried by 30 motor transport lorries is 6 tons. Two more large loaded lorries, one of 20 tons and the other of 45 tons, are added to the former convoy. What is the average tonnage now?

(14) In measuring the diameter of a steel rod, the following readings were taken at intervals of 6 inches: ·231″, ·232″, ·23″, ·234″, ·23″. Find the average of the five readings.

(15) In 3 months' earnings, 15 men receive £51 7*s.* 6*d.* each and 9 women obtain £26 12*s.* 6*d.* each. What is the average amount received by the 24 hands?

(16) A casting is about to be machined up in a lathe. The diameters at six points in its length are read as: $11\frac{1}{4}''$, $11\frac{3}{8}''$, $11\frac{1}{8}''$, $11\frac{1}{4}''$, $11\frac{7}{16}''$, $11\frac{3}{8}''$. The machined diameter is 11 inches. What is the average depth of cut?

(17) Out of 100 hands in a workshop three were selected to do a small job. The times in which the 3 hands did the job were 5·34, 5·38, and 5·33 seconds respectively. Take the average performance of these three hands to be the average of the whole 100, and find the time required to produce 10 million of the job by the 100 hands.

(18) Three men on piece work earn an average of 17*s.* 2½*d.* each in one day. The first man earns 18*s.* and the second man earns 1*s.* 9*d.* less than the first. What does the third man earn?

CHAPTER IV

SYMBOLIC EXPRESSION

To assist an engineer and others in calculations, it is often found convenient to employ letters to represent some number the numerical value of which is not specified, but which, in some cases, may have to be found.

A letter employed in this way is called a "**symbol.**"

Example (1). *When a certain number is multiplied by* 5, *and* 3 *is then added to the result, the total is* 38. *Find the number.*

The problem may be expressed thus:

$$5 \times \text{required number} + 3 \text{ is equal to } 38.$$

Now let the required number be represented by the letter x, then the problem can be expressed more concisely, thus:

$$5 \times x + 3 = 38.$$

The multiplication sign is usually omitted between a letter thus used and a number multiplying it, so that $5 \times x$ is written as $5x$. Consequently the problem more concisely stated is:

$$\begin{aligned} 5x + 3 &= 38. \\ \text{But } 35 + 3 &= 38, \\ \text{hence } 5x &= 35, \\ \text{from which } x &= 7. \end{aligned}$$

Thus the "unknown" number is found to be 7.

Formulæ

An expression of a mathematical truth by symbols is called a "formula."

Example. The circumference of a circle is the complete length of the curve, and the diameter is the length of the straight line passing through the circle centre and intercepted by the circumference.

If the student takes a thin thread and wraps it round a circular rod exactly once, then measures the length of this thread, the result will be the length of the circumference. If this length is divided by the diameter, the result should always be the same.

$$\text{Thus } \frac{\text{circumference}}{\text{diameter}} = \text{same amount.}$$

The letter which is always used, to represent this "same amount" is taken from the Greek alphabet, and is called pi. The symbol is π.

Hence the *circumference of a circle may be stated as equal to its diameter multiplied by* π.

$\therefore$ If D be the length of the diameter, and C be the circumference, then $\mathbf{C} = \pi\mathbf{D}$, which is called a formula relating the circumference to the diameter.

Examples of the building up of Formulæ by Symbolic Expressions.

(1) The cutter of a milling machine is d inches in effective diameter and makes N revolutions a minute. State the cutting speed in feet per minute by a formula.

$$\text{Circumference of cutter} = \frac{\pi d}{12} \text{ ft.}$$

$$\text{Cutting speed} = \frac{\pi d}{12} \text{ N ft. per min.}$$

(2) If a bolt weighs w lb., give a formula for the number of bolts in one hundredweight.

Let N = number of bolts.
As 112 lb. = one hundredweight,

$$\text{then N} = \frac{112}{w}.$$

(3) From a steel rod, n pieces of L inches each in length are cut off. The original length of the rod is M feet. Give a formula for the amount left in feet.

$$\text{Length cut off} = \frac{n\text{L}}{12} \text{ ft.}$$

$$\therefore \text{Remainder} = \mathbf{M} - \frac{n\mathbf{L}}{12}.$$

(4) If L feet of a steel rod weighs W lb., find the weight of n feet.

$$\text{Weight of one foot} = \frac{\text{W}}{\text{L}} \text{ lb.}$$

$$\text{Weight of } n \text{ feet} = \frac{\mathbf{W} \times n}{\mathbf{L}} \text{ lb.}$$

(5) How many truck loads of w cwt. each are in a heap of scrap metal weighing T tons ?

Let n = number of truckloads. T tons = 20 T cwts.

$$\text{Then } n = \frac{20\text{T}}{w}.$$

(6) An object falling freely through space increases its velocity by 32·2 feet per second, every second. Give a formula for its speed at the end of n seconds from the start.

Speed = **32·2n feet** per second.

(7) In Example (6) how far does the object fall in n seconds? Since the object commences from rest and increases its speed uniformly to $32{\cdot}2n$ feet per second, the average speed during the n seconds is:

$$\text{Average speed} = \frac{32{\cdot}2n}{2} \text{ feet per second.}$$

$$\therefore \text{distance travelled} = \frac{32{\cdot}2n}{2} \times n.$$

$$= 16{\cdot}1n^2.$$

(8) Divide a steel rod L feet long into two parts such that one part is $\frac{3}{4}$ the length of the other part.

Let x feet be the length of the longer part.

Then $\frac{3}{4}x$ is the length of the shorter part.

$$\therefore x + \tfrac{3}{4}x = L,$$

$$i.e.\ 1\tfrac{3}{4}x = L$$

$$x = \frac{L}{1\frac{3}{4}} \text{ feet} = \tfrac{4}{7}\text{ L feet.}$$

$$\text{The other part} = \tfrac{3}{7}\text{ L feet.}$$

(9) Two men, A and B, earn 23*s*. in one day, but A earns 5*s*. more than B. State these quantities by formula.

Let A equal A's amount and let B equal B's amount.

Then $A + B = 23$.

And $A - B = 5$.

Hence $A + B + A - B = 28$. From which $\mathbf{A} = 14s.$

$\therefore 2A = 28$. and $\mathbf{B} = 9s.$

EXERCISE 4

(1) A certain number is multiplied by four and twelve is then added to the product. The result is equal to 36. State this as a formula. Also find the original number.

(2) A certain number is multiplied by nine, and the product is subtracted from 108. The result is = 18. State this as a formula, then find the value of the original number.

(3) A number is divided by eleven and the quotient is added to 51. The result is 203. State this as a formula and then calculate the original number.

(4) A man and some boys earned 17 shillings for a certain job: the man gave 1*s*. 6*d*. to each boy and then had 5*s*. for himself. State this as a formula and then find the number of boys there were.

(5) The circumference of a circle is eleven inches. Let D be the diameter, and state the formula. Now find the length of the diameter. (Use $\frac{22}{7}$ as π.)

For further problems on Symbolic Expression, see exercise in Algebra, Chapter X.

CHAPTER V

SQUARES AND SQUARE ROOTS

THE **square of a number** is the product of the number multiplied by itself.

Thus $3 \times 3 = 9$. This is the square of 3.
,, $12 \times 12 = 144$. ,, ,, ,, ,, ,, 12.
Likewise $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$. ,, ,, ,, ,, ,, $\frac{3}{4}$.
,, $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. ,, ,, ,, ,, ,, $\frac{1}{2}$.

The reader should note carefully that when any fraction in which the denominator is greater than the numerator is squared, the value of the result is less than that of the original fraction.

Similarly, in the case of decimal fractions :

$$\cdot 9 \times \cdot 9 = \cdot 81.$$
$$\cdot 6 \times \cdot 6 = \cdot 36.$$

Let **N** be a number : it is usual to represent the square of N by the symbol $\mathbf{N^2}$—the small "*2*" at the top corner of N meaning that the number is the product of **two N's.**

Thus in the cases above, 3×3 and 12×12 may be written 3^2 and 12^2 respectively.

In the case of a fraction, the whole of the fraction should be in a bracket thus : $(\frac{3}{4})^2$ and $(\frac{1}{2})^2$, meaning that both numerator and denominator are squared.

Powers of Numbers

The **square** of a number is often called the **second power** of the number, likewise the **cube** of a number is called the **third power,** and so on.

Thus $3^3 = 3 \times 3 \times 3 = 27$.
,, $12^3 = 12 \times 12 \times 12 = 1728$.
Similarly $(\frac{3}{4})^3 = \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64}$.
,, $\cdot 9^3 = \cdot 9 \times \cdot 9 \times \cdot 9 = \cdot 729$.

The small figures placed over numbers, as in the examples above, are called "indices," which is the plural of index.

Square Root

We have seen above that 9 is the "square" of 3. Then 3 is called the "*square root*" of 9.

Again, 144 is the square of 12. Then 12 is called the square root of 144.

A square root is indicated by the sign "$\sqrt{}$" which is written thus $\sqrt{9}$.

$$\text{Hence } 3 = \sqrt{9}$$
$$\text{and } 12 = \sqrt{144}$$
$$\text{Similarly, since } \left(\tfrac{3}{4}\right)^2 = \tfrac{9}{16}$$
$$\text{then } \tfrac{3}{4} = \sqrt{\tfrac{9}{16}}$$
$$\text{and since } (\cdot 9)^2 = \cdot 81$$
$$\therefore \cdot 9 = \sqrt{\cdot 81}$$

Similarly with **cubes** and **cube roots**—the cube of 12 = 1728, and the cube root of 1728 is 12. The sign $\sqrt[3]{}$ indicates cube root. Thus $3^3 = 27$ and $3 = \sqrt[3]{27}$.

To Find Square Roots

In workshop calculations the roots of numbers are usually not so easily obtained as in the cases quoted above. The following examples illustrate the method to be adopted.

Example (1). *Find the square root of* 1711·4769.

First divide the number into pairs of figures away from the decimal point.

First step. Place in the answer the highest number which, when squared, is not greater than 17, *e.g.* 4. Square the 4 and subtract from 17.

Multiply the answer by 2, i.e. 8. The next figure which goes in the answer is placed next to this 8, *e.g.* 81.

Again multiply the answer by 2. Result is 82. Now, what figure when placed at the end of 82 will go so many times into 3047? It is 3, thus 823.

Repeat the above process. Again get twice the answer : 826. Add the next figure which will go in the answer : 8267.

```
17,11·47,69(41·37
16
---
 111

  81
------
  3047



  2469
    57869


    57869
   ------
    .....
```

Bring down the next pair of figures and add to the remainder.

Then 81 is placed under the 111 and subtracted, since it will only go once into 111.

The next pair of figures to be brought down are decimals, hence place the decimal point in the answer. Bring down the 47.

Place 3 in the answer. Then 823 × 3 is placed under the 3047 and subtracted.

Bring down the next pair of figures and put them to the end of 578.

Multiply 8267 by 7 and subtract.

Thus 41·37 is the square root of 1711·4769.

As the student may at times require to get the square root of a number with great accuracy, he is advised to master the above method, although an easier method will be described later : *i.e.* the logarithmic method. Calculations with the aid of logarithms are discussed in Book 2 of this subject.

Example (2). Find $\sqrt{92416\cdot}$

In this case there is an odd figure on the extreme left of the decimal point after taking pairs of figures from the decimal point, so start with this figure. The first figure to go in the answer is 3. Bring down the next pair of figures, *i.e.* 24. The next figure in the answer is 0. Bring down the next pair of figures, *i.e.* 16. Place 4 following the 60, and get 604, since 4 is the next figure in the answer.

$$\begin{array}{r|l} & 9,24,16\cdot(304 \\ & \underline{9} \\ 60 & .\overline{2416} \\ 604 & \underline{2416} \\ & .... \end{array}$$

Example (3). Find $\sqrt{\cdot 0791634}$.

In this case there is a decimal fraction only, but still divide into pairs of figures from the decimal point, and commence with the first pair of figures. First, place the decimal point in the answer,

$$\begin{array}{r|l} & \cdot 07,91,63,4(\cdot 28136 \\ & \underline{4 \quad \cdot} \\ 48 & 391 \\ & \underline{384} \\ 561 & ..763 \\ & \underline{\quad 561} \\ 5623 & \quad 20240 \\ & \underline{\quad 16869} \\ 56266 & \qquad 337100 \\ & \qquad 337596 \end{array}$$

In this case the number given is not a perfect square, so the answer must be continued to a required number of decimal places. In this case the answer is given to the nearest $\frac{1}{100000}$th.

Example (4). Obtain $\sqrt{\cdot 000745623}$.

$$\begin{array}{r|l} & \cdot 00,07,45,62,3(\cdot 027306 \\ & \underline{\quad 4} \\ 47 & \quad 345 \\ & \underline{\quad 329} \\ 543 & \qquad 1662 \\ & \underline{\qquad 1629} \\ 5460 & \qquad\quad 333000 \\ & \underline{\qquad\quad 327636} \\ 54606 & \qquad\qquad \cdot 5364 \end{array}$$

Answer is ·027306 to the sixth decimal place.

✓EXERCISE 5

Find the squares of the following numbers:

(1) 25. (2) 13. (3) 2·1. (4) 1·2.
(5) 5·27. (6) ·78. (7) 10·01. (8) 64·8.

Extract the square roots of the following exercises:

(9) $\sqrt{64}$. (10) $\sqrt{82{\cdot}2649}$. (11) $\sqrt{3{\cdot}9601}$. (12) $\sqrt{{\cdot}160801}$.
(13) $\sqrt{\frac{49}{64}}$. (14) $\sqrt{2\frac{1}{4}}$. (15) $\sqrt{12\frac{1}{4}}$. (16) $\sqrt{2\frac{7}{9}}$.

(First express the last three as improper fractions.)

CHAPTER VI

RATIO AND PROPORTION

Ratio

In the ordinary circumstances of everyday life one commonly makes statements which involve the idea of comparison of two quantities in a form which is known as a ratio. Thus it might be said that the production of a particular job by a machine is three times what it used to be. This may be expressed by saying that the amounts produced are in the **ratio** 3 is to 1. This is commonly written 3 : 1, or as a fraction.

Mostly, engineering drawings are said to be in a reduced scale, say, $\frac{1}{4}$ size. What we really mean by this is that any particular dimension on the drawing is to the actual length on the machine in the ratio 1 is to 4; i.e. 1 : 4. The two dots, one under the other, represent the words "is to." The student has previously observed that the sign (÷) means division, the sign (:) means division with the horizontal line omitted.

Thus the ratio of 24 to 3 means the number of times 3 is contained in 24, i.e. it is the fraction $\frac{24}{3}$ which is equal to 8. Note that the first number stated is in the numerator and the second number is the denominator of the fraction.

The fraction $\frac{2}{3}$ means, "**the ratio of 2 to 3.**"

The ratio of 7 to 8 is expressed by the fraction $\frac{7}{8}$.

The ratio of 3 to 24 is expressed by the fraction $\frac{3}{24}$.

This latter fraction may be reduced to $\frac{1}{8}$ by cancelling the common factor 3.

Practical illustrations of ratios.

(1) If a drawing of a machine piece is done to a scale of one inch to a foot, then every dimension on the drawing is to the similar dimension on the machine piece, in the ratio of $\frac{1}{12}$.

(2) In the figure below, BC is 1″ long and AB is 5″, that is the two lengths are in the ratio 1 : 5.

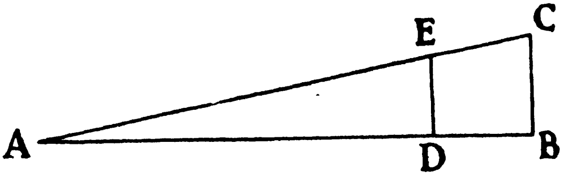


FIG. 19.

The student will find on measuring that any other vertical such as DE, and horizontal AD, are in the same ratio, i.e. 1 : 5, or $\frac{1}{5}$.

This principle is made use of in the construction of a particular kind of engineers' scale, called "**The Diagonal Scale.**" See construction of scale, Fig. 20.

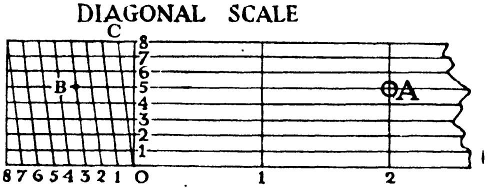


FIG. 20.

Further explanation of the construction of the diagonal scale is shown below, page 32, and by Fig. 23.

Proportion

The idea of direct proportion can be illustrated by Fig. 19, for the ratio of DE to AD is equal to the ratio BC to AB. This statement is commonly written—DE : AD :: BC : AB. As before stated, the sign (:) means (is to) and the sign (::) means (as).

Sometimes the sign (::) is replaced by the sign (=), i.e. equals. Then the proportion would be written—DE : AD = BC : AB, or, in fractional form, thus: $\frac{\text{DE}}{\text{AD}} = \frac{\text{BC}}{\text{AB}}$

If one quantity M depends on another quantity N in such a way that if M is increased or decreased in a given ratio, N increases or decreases in the same ratio, then N is said to be directly proportional to M.

Example (1). *Bronze is an alloy of copper and tin. Two-thirds of a certain bronze is copper, how much copper is in* 150 *lb. of this bronze?*

Let b = the amount of copper.

Then $b : 150 = 2 : 3$ or $\frac{b}{150} = \frac{2}{3}$. $\therefore b = \frac{2}{3} \times 150$.

$b = 100$ lb.

Example (2). *Fig.* 21 *shows a tapered rod. Find the amount of taper per inch and also taper per foot length.*

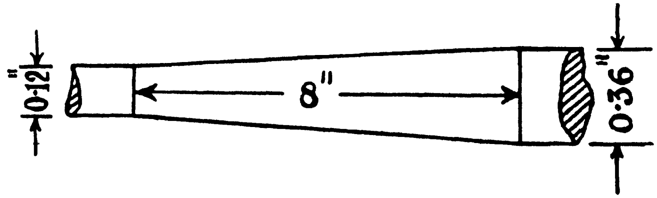


FIG. 21.

Taper on 8" is 0·36" — ·12" = 0·24"

„ „ 1" „ $= \frac{0{\cdot}24''}{8} = 0{\cdot}03''$.

„ „ 12" „ $= 0{\cdot}03'' \times 12 = 0{\cdot}36''$.

Example (3). *The taper in the following reamer shank* (Fig. 22) *is* $\frac{1}{4}''$ *per foot length. From* A *to* B *is* 4". *If the diameter at* A *is* $\frac{5}{8}''$, *what should be the diameter at* B ?

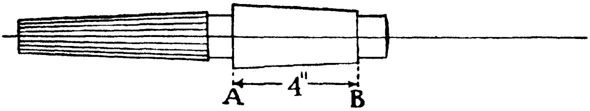


FIG. 22.

The taper is on 4" length, hence the difference in diameter between B and A is $\frac{4}{12} \times \frac{1}{4} = \frac{1}{12}''$.

$\therefore$ Diameter at B $= \frac{5}{8} - \frac{1}{12} = \frac{13}{24}''$.

Construction of the Diagonal Scale

In Fig. 23 AB represents 1", and it is divided into eighths. BC represents $\frac{1}{8}''$.

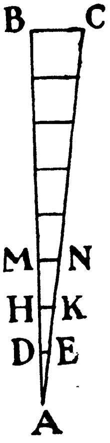


FIG. 23.

The ratio BC : AB is as $\frac{1}{8} : 1$, *i.e.* $= \frac{1}{8}$.

Now DE : AD = BC : AB,

$$i.e.\ \frac{DE}{AD} = \frac{BC}{AB},$$

But BC : AB $= \frac{1}{8}$.

$$\therefore \frac{DE}{AD} = \frac{1}{8},\ i.e.\ DE = \frac{1}{8}\,AD.$$

Since AD is $\frac{1}{8}''$, DE $= \frac{1}{64}''$.

In a similar manner HK, MN, etc., may be shown to be $\frac{2}{64}$, $\frac{3}{64}$, etc., respectively.

The Diagonal Scale

The student will observe that the triangle OC8 on the diagonal scale Fig. 20 is divided like the triangle ABC Fig. 23, that is, the vertical column of numbers represents $\frac{1}{64}''$. The horizontal distance between each pair of sloping lines is $\frac{1}{8}''$. The reading from A to B is $2'' + \frac{3}{8}'' + \frac{5}{64}'' = 2\frac{29}{64}''$. Other readings are read similarly.

Inverse Ratio

It has been previously noted that the ratio of 7 to 8 may be expressed by the fraction $\frac{7}{8}$. The inverse of this ratio is the reciprocal of the fraction: i.e. $\frac{1}{\frac{7}{8}} = \frac{8}{7}$. Thus the inverse of a fraction is obtained by interchanging numerator and denominator.

Inverse Proportion

When a quantity M *depends on another quantity* N *in such a way that if* M *is increased in any given ratio, then* N *decreases in the same ratio, then* M *is said to vary inversely as* N, *that is* M *is inversely proportional to* N.

Many aspects of inverse proportion occur in engineering:

Example (1). For a particular cutting speed, either in a centre lathe or a milling machine, the greater the diameter of the work or cutting tool, the less will be the revolutions per minute required.

$$\text{i.e. Revolutions per minute} = \frac{\text{cutting speed}}{\frac{22}{7}D},$$

where D is the diameter of the work or of the cutter respectively.

Hence the number of revolutions per minute varies inversely as the diameter.

Example (2). In the case of a belt or rope drive, Fig. 24, πx is the circumference of the small pulley.

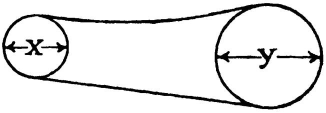


FIG. 24.

Let n be the revolutions per minute of this pulley, then $n\pi x$ is the speed of the pulley rim and the belt in feet per minute.

Similarly for the bigger pulley rim, where N is its revolutions per minute.

$N\pi y$ is the speed of the rim in feet per minute.

Since the belt covers both pulley rims, the speed is the same:

$$\therefore n\pi x = N\pi y.$$

Cancel the common factor π.

$$\text{Then } nx = Ny.$$

$$\text{i.e. } \frac{n}{N} = \frac{y}{x}.$$

That is, the ratio of the revolutions per minute of the two pulleys is in the inverse ratio to the two diameters.

Example (3). In the expansion of air and other gases, one law of expansion is such as :

$$P = \frac{2000}{V},$$

in which P is pressure, V is the volume.

From this law the student will easily observe that as V increases the value of P becomes less. That is :

P is inversely proportional to V.

Let $V = 1, 2, 4, 5$

then $P = 2000, 1000, 500, 400$, respectively.

Example (4). *A belt pulley is* $3\frac{1}{2}'$ *diameter, and makes* 90 *revolutions a minute. If this belt has to drive a pulley at* 270 *revolutions a minute, what must be the diameter of the pulley ?*

$\frac{22}{7} \times 3\frac{1}{2} \times 90$ = speed of the belt in feet per minute.

Let *d* be the diameter pulley.

$$\therefore \frac{\cancel{22}}{\cancel{7}} \times 3\tfrac{1}{2} \times \cancel{90} = \frac{\cancel{22}}{\cancel{7}} \times \overset{3}{\cancel{270}} \times d.$$

Cancelling common factors : $3\frac{1}{2} = 3d$.

$\therefore d = 1'\ 2''$.

One quantity may be Proportional to the Square of another

Thus the kinetic energy stored in a moving object is proportional to the square of the velocity in feet per second.

The formula is :

$$\text{Kinetic energy} = \frac{W \times V^2}{2g}.$$

Where W is the weight, V is the velocity in feet per second and g is 32·2.

If W = 200 lb. and V = 16, then the kinetic energy is 795 ft.-lb.

A quantity may be Proportional to the Square Root of another

Thus in the formula—

$$T = 2\pi \sqrt{\frac{L}{g}},$$

T is the time in seconds of a vibration in a simple pendulum, $\pi = 3{\cdot}142$, L is the length of the pendulum in feet and $g = 32{\cdot}2$.

If L = 3·26 feet, T = 2 seconds.

Proportional Parts

It is often necessary to divide a quantity into parts proportional to given numbers.

Example (1). *Thus three men* X, Y, *and* Z *produced* 19 *articles between them.*

X *produced* 9 *articles.*
Y ,, 6 ,,
and Z ,, 4 ,,

The amount of wages paid for the total production was £4 5*s*. 6*d*., *how much should each man receive?*

The rule is to add all the articles produced together, *i.e.* 9 + 6 + 4 = 19. Divide the money by 19: this will give the cost of producing one article. Then multiply the result by 9, 6, and 4, respectively.

Thus the share for X is $\left(\frac{\pounds 4\ 5s.\ 6d.}{19}\right) \times 9.$

,, ,, ,, ,, Y ,, $\left(\frac{\pounds 4\ 5s.\ 6d.}{19}\right) \times 6.$

,, ,, ,, ,, Z ,, $\left(\frac{\pounds 4\ 5s.\ 6d.}{19}\right) \times 4.$

i.e. £2 0*s*. 6*d*., £1 7*s*., and 18*s*. respectively.

Example (2). *Three men* X, Y, *and* Z *enter into a partnership,* X *invests* £1,080, Y, £450, *and* Z, £900. *If the profit over a period was* £202 10*s*., *what should each receive?*

Sum of the investments is £2,430.

Profit per £1 investment $= \frac{\pounds 202\ 10s.}{2430}.$

The share for X $= \frac{\pounds 202\ 10s.}{2430} \times 1080.$

,, ,, ,, Y $= \frac{\pounds 202\ 10s.}{2430} \times 450.$

,, ,, ,, Z $= \frac{\pounds 202\ 10s.}{2430} \times 900.$

i.e. £90, £37 18*s*., and £75, respectively.

Example (3). *A certain brass is made from copper and zinc, such that the proportion of copper to zinc is* 5 *to* 2. *Find the weight of copper and zinc required to make* 28 *cwt. of this brass.*

There are 5 parts of copper to 2 of zinc—*i.e.* 7 parts in all.

$\therefore \frac{28}{7} \times 5$ cwt. is the amount of copper.

$\frac{28}{7} \times 2$ cwt. is the amount of zinc.

i.e. 20 cwt. and 8 cwt. respectively.

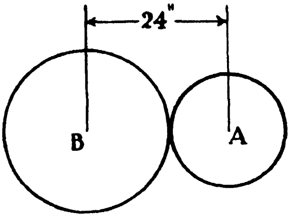


FIG. 25.

Example (4). *Two tooth wheels, Fig. 25, centres B and A, have their pitch diameters in the ratio 3 : 2. The distance between centres of A and B is 24". Calculate the pitch diameters of the two wheels.*

As the pitch diameters are in the ratio 3 : 2, so are the pitch radii.

The sum of the pitch radii is 24".

∴ pitch radius of B $= \frac{3}{5} \times 24 = 14{\cdot}4''$.

and „ „ „ A $= \frac{2}{5} \times 24 = 9{\cdot}6''$.

Example (5). In the right-angled triangle shown in Fig. 26, if the student will measure the lengths of the lines, he will observe that the following ratios are numerically accurate :

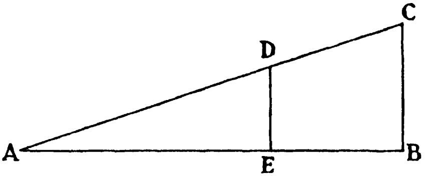


FIG. 26.

$$\frac{BC}{AB} = \frac{ED}{AE}. \qquad \frac{BC}{AC} = \frac{ED}{AD}.$$

$$\frac{AE}{AD} = \frac{AB}{AC}. \qquad \frac{AD}{DC} = \frac{AE}{EB}.$$

The student will observe later (see Chapter XIII) that ratios of this kind are basic in the work for all trigonometrical calculations.

EXERCISE 6

(1) If $\frac{x}{20} = \frac{2}{5}$, what is the value of x ?

(2) The traverse of a lathe tool is $\frac{1}{16}''$ per revolution of the work. How many revolutions will be made for the traverse to be $4\frac{3}{4}''$?

(3) In question (2), if the number of revolutions of the work per minute is 114, what length of time is needed to traverse the $4\frac{3}{4}''$?

(4) The pitch of a screw thread is $\frac{1}{12}''$. How many threads are there in a screw $4\frac{1}{2}''$ long ?

(5) The pitch of a screw is 1·27 millimetres. How many threads are there in a screw 4" long ? (There are 25·4 millimetres in one inch.)

(6) Two pulleys are connected by a belt. The driver is 18" diameter and the driven pulley is 54" diameter. The driver makes 240 revolutions

per minute. What revolutions per minute does the driven pulley make ?

(7) Two tooth wheels are in mesh. The driver has 25 teeth and makes 120 revolutions per minute. How many teeth must the driven pulley have so as to make 30 revolutions a minute ?

(8) The pitch of a lathe leading screw is ¼". If the screw makes 15 revolutions a minute, how much is the traverse per minute ?

(9) The number of revolutions per minute of a milling cutter 3½" diameter is 105. Calculate the speed of the cutter edge in feet per minute.

(10) The blue-print of a machine piece is to a scale of 1 to 4. The length of a steel link on the blue-print is 6¾". What is the actual length of the link ?

(11) In a particular bronze there are two parts of copper to one part tin. How much of each metal is required to make 45 cwt. of the bronze ?

(12) Sixty-four pounds of air at ordinary temperature and pressure occupy 714 cubic feet. What would be the volume of 24 lb. of air ?

(13) Three partners *x*, *y*, and *z* invest money in a business—*x* £1,700, *y* £1,000, and *z* £500. If the profit over a period of time is £352, what should be the share of each ?

(14) A particular white metal for bearings contains 21 parts tin, 3 parts copper, and 1 part antimony. How much of each metal is in 1 cwt. of the white metal ?

(15) If $\frac{W}{100} = \frac{4}{75}$, find the value of W.

(16) The rate for a certain job is 15*s*. for 100 articles. What should one receive for the production of 420 articles ?

(17) In the building of a locomotive, 90 parts of it is steel, 8 copper, and 2 other metals. Find the weight of each metal in a locomotive weighing 54 tons.

(18) A uniform lever is 20" long and the fulcrum is at the centre. A load of 30 lb. is hung at a point 8 inches from the fulcrum. Where must a load of 50 lb. be hung to give balance to the lever ? (Hint : The weights are inversely proportional to their distances from the fulcrum.)

(19) A piece of metal weighing 40 lb. is bolted to the face-plate of a lathe 9" from the centre. A weight of 45 lb. is handy. At what position on the face-place must it be bolted in order to give balance ? (Hint : Treat as a lever.)

(20) Draw a right-angled triangle with the sides, including the right angle, 4½" and 6" long. Measure the length of the hypotenuse and calculate the ratios of the lengths of the former two sides with the hypotenuse.

(21) A machine piece has a reciprocating motion and makes 2800 strokes of 4½" in the minute. Calculate the average speed in feet per minute.

(22) The weight of various lengths of steel rod of the same diameter are given in the table :

Weight, lb.	1·8	4·5	7·2	10·25	15
Length, inches	3	7½	12	18·75	25

One of the weights is wrong, which one is it, and what should it be ?

(23) A metallurgist added a quantity of chromium to a weight of steel so that the chromium added was 5 per cent. of the steel. There was then 35 tons of the chrome steel formed. What was the weight of the original steel and the weight of chromium added ?

(24) The cutting tool of a lathe advances $\frac{1}{8}''$ for a complete revolution of the handle. What fraction of a revolution is required for a feed of $\frac{3}{64}''$?

(25) How many minutes will be required to drill a hole 4″ deep with a constant feed of ·015″ per revolution ? Revolutions of drill per minute 200.

(26) The gradient of a works wagon track is a rise of one in a length of 5½ along the track. What is the rise for $\frac{1}{16}$ of a mile ?

(27) A machined part is listed in an American catalogue at 142½ dollars per dozen. What would 5 dozen cost in English money if the rate of exchange were £1 = 4·755 dollars ?

(28) A piece of metal, weight 60 lb., is bolted to the face-plate of a lathe 8″ from the centre. Where must a weight of 32 lb. be placed to cause balance ?

CHAPTER VII

THE METRIC SYSTEM

Measurement of Length

In the metric system the *metre* is the unit of length, and the following table gives the names of the other units:

Kilometre = 1,000 metres.
Hectometre = 100 metres.
Decametre = 10 metres.
metre
decimetre = $\frac{1}{10}$ or ·1 metre.
centimetre = $\frac{1}{100}$ or ·01 metre.
millimetre = $\frac{1}{1000}$ or ·001 metre.

The **metre is 39·37 inches,** and it was considered that the distance round the equator was 40 million metres.

The metric system is based upon decimal fractions, and the rules of decimals apply to calculations with them.

Example (1). *Convert* 2·7234 *Kilometres to metres.*

Since there are 1,000 metres in a Kilometre, multiply 2·7234 by 1,000. The result is 2723·4—the decimal point is moved three places to the right as shown on p. 9.

Example (2). *Convert* 234·6 *millimetres to centimetres.*

Since there are 10 millimetres in one centimetre, move the decimal point one place to the left—i.e. divide by 10. Answer 23·46 centimetres.

Example (3). *Find the number of millimetres in an inch.*

From the table above we note that there are 100 centimetres in one metre. Also there are 39·37″ in one metre.

∴ 39·37″ are equal to 100 centimetres.

Hence 1″ is equal to $\frac{100}{39 \cdot 37}$ centimetres.

I.e. 1″ is equal to 2·54 cm.

From this we get that 1″ = 25·4 mm.

Example (4). *Two dimensions on a drawing were given as* 45 *mm. and* 70 *mm. Convert these dimensions to inches correct to the nearest hundredth of an inch.*

Since 25·4 mm. = 1″ (Examples)

$$\frac{45}{25{\cdot}4} = 1{\cdot}77''.$$

$$\frac{70}{25{\cdot}4} = 2{\cdot}76''.$$

Example (5). *Convert* 10 *cm. to inches.*

$$\frac{10}{2{\cdot}54} = 3{\cdot}937''.$$

Example (6). *The diameter of a cylinder is given as* 75·6 *mm. Convert this to inches.*

$$\frac{75{\cdot}6}{25{\cdot}4} = 2{\cdot}97''.$$

Example (7). *An aeroplane has a speed of* 250 *miles an hour. Convert this speed to Kilometres per hour.*

One mile = 5280 × 12″.

One Kilometre = 39·3708 × 1000 = 39370·8″.

$\therefore$ number of Kilometres in one mile $= \dfrac{5280 \times 12}{39370{\cdot}8}$

$= 1{\cdot}609.$

Hence speed of the aeroplane $= 1{\cdot}609 \times 250$

$= \mathbf{402\tfrac{1}{4}}$ **Km. per hour.**

British Association Standard Thread (B.A.)

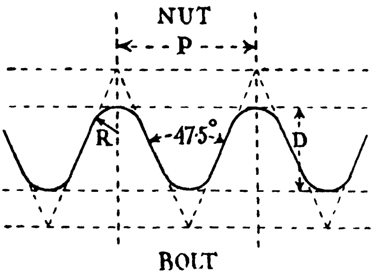


FIG. 27.

The angle of the B.A. Standard Thread is 47½°, and it is rounded off equally both top and bottom with a radius practically equal to $\frac{2}{11}$ths of the pitch, leaving the actual depth of the thread equal to ·6 of the pitch. The diagram, Fig. 27, gives the form and formula for its origin.

P = Pitch.

$R = \frac{2}{11}P.$

$D = {\cdot}6P.$

Example (1). For bolt 6 mm. diameter.

Pitch = 1 mm.

$R = \frac{2}{11}$ mm.

$D = {\cdot}6$ mm.

Example (2). For bolt 2·5 mm. diameter.

$$P = \cdot 48 \text{ mm.}$$
$$R = \tfrac{2}{11} \times \cdot 48 = \cdot 087 \text{ mm.}$$
$$D = \cdot 6 \times \cdot 48 = \cdot 288 \text{ mm.}$$

Abbreviations

Abbreviations are used as follows :

cm. is used for centimetre.
Km. ,, ,, ,, Kilometre.
mm. ,, ,, ,, millimetre.
Kgm. is used for Kilogram
c.c. ,, ,, ,, cubic cm.
gms. ,, ,, ,, grams.

Weight

The **gramme** (gram) is the unit of weight, and this unit has the same multiples and subdivisions as the metre. The most important of these are the **Kilogram** and the **milligram.**

The Kilogram = 1000 grams.
The milligram = $\frac{1}{1000}$ gram.

One Kilogram = 2·2046 *lb., i.e.* approximately $2\frac{1}{5}$ lb.

Example (1). *Convert* 100 *Kilograms to lb.*

100 Kilograms = 2·2046 × 100 lb.
= **220·46 lb.**

Example (2). *Convert* 3·1 *Kilograms to lb.*

3·1 × 2·2046 = **6·83426 lb.**

Example (3). *How many Kilograms are there in* 3 *cwt.?*

3 cwt. = 336 lb.

$$\text{Kilograms} = \frac{336}{2 \cdot 2046} = \mathbf{150 \cdot 41.}$$

Capacity

The unit of capacity in the metric system is the **Litre.** This unit is equal to 1000 cubic centimetres.

The gram weight was fixed as the weight of a cubic centimetre of water, consequently the weight of a litre of water is one Kilogram.

Example (1). *Find the weight in lb. of* 2000 *litres of water.*

The weight is 2000 Kilograms.
One Kilogram = 2·205 lb.
∴ the weight of 2000 litres of water = 2000 × 2·205 lb.
= **4410 lb.**

Example (2). *How many litres of water weigh a ton ?*

One litre of water = one Kilogram in weight.
One Kilogram = 2·205 lb.

$$\therefore \text{litres of water in 2240 lb.} = \frac{2240}{2 \cdot 205} = 1016.$$

Table of Equivalents

1 inch = 2·54 centimetres
1 mile = 1·609 Kilometre
1 lb. = 0·4536 Kilogram
1 cubic inch = 16·39 c.c.
1 gallon = 4·546 litres
1 square inch = 6·452 sq. cm.
1 cubic foot = 28·317 litres
1 cubic yard = 0·76336 cubic metre
1 ton = 1016 Kilogram
1 yard = ·914399 metre

1 centimetre = 0·3937 inch
1 Kilometre = 0·6214 mile = ⅝ mile approx.
1 Kilogram = 2·205 lb.
1 c.c. = 0·0610 cubic inches
1 litre = 0·220 gallon
1 sq. cm. = 0·1550 square inch
1 litre = 1·7598 pints
1 cubic metre = 35·3148 cubic feet
1000 Kilogram = ·9842 ton
1 metre = 39·37 inches

✓ EXERCISE 7

✓ (1) Convert 20 centimetres to inches.

(2) Convert 5300 metres to Kilometres.

(3) Two dimensions on a blue-print were 45 mm. and 280 mm. Convert these dimensions to inches.

(4) In 35·2 lb., how many Kilograms are there ?

(5) The cutting speed at a lathe is 90′ per minute. Convert this to metres per minute.

(6) Express in inches 30, 52, 50, and 72 cm.

(7) Which is the smaller, 8″ or 22 cm. ?

(8) How many litres are equivalent to 3½ gallons ?

(9) An oil tank has a capacity of 10 gallons. What is the equivalent capacity in litres ?

(10) Petrol weighs 8 lb. per gallon. What is the weight of 91 litres ?

(11) A bolt has 12·7 threads per centimetre length of the bolt. Transform and state the number of threads per inch length.

(12) An object weighs 5½ lb. Convert this weight to grams.

(13) Five litres of oil weigh 44 lb. Find its weight in lb. per gallon.

(14) For a certain feed, the cutting speed for a particular tool tip is stated to be 250 feet per minute. Transform to a speed of metres per second.

(15) A 10-gallon drum of oil was ordered, but a 10-litre drum was delivered. How many gallons were short in the delivery ?

(16) The cutting force at a tool point is 1300 lb. Convert this force to Kilograms.

(17) The shear strength of a certain metal is 19 tons per sq. inch. State the strength in Kilograms per sq. centimetre.

(18) An operative is instructed to work a job to the dimension $5{\cdot}5\begin{smallmatrix}+\cdot002\\-\cdot002\end{smallmatrix}$ cms. Convert the dimensions to the inch scale. State the "tolerance" allowed both on the metric and inch scales.

CHAPTER VIII

PERCENTAGES

A CONVENIENT way of comparing ratios and fractions is to express them in the form of fractions equal to them but having 100 as denominator.

For example, $\frac{3}{4}$ can be expressed in the equivalent form $\frac{75}{100}$. When that is done it can be said that $\frac{3}{4}$ is 75 hundredths, or 75 **per cent.**

Similarly, $\frac{21}{25}$ can be expressed as $\frac{84}{100}$, that is 84 hundredths or 84 **per cent.**

It is then much easier to compare $\frac{3}{4}$ with $\frac{21}{25}$.

This is what is meant when a fraction is expressed as a **percentage.**

Conversely, 28 per cent. of a mixture means $\frac{28}{100}$ of the mixture, and 5 per cent. of a sum of money means $\frac{5}{100}$ of it.

The symbol % is usually employed to express "per cent." Thus,

5% of £40 means $\frac{5}{100}$ of £40.

When we say that interest on money in the Post Office Savings Bank is $2\frac{1}{2}$%, we mean that $\frac{2\frac{1}{2}}{100}$ of the money invested is paid as interest each year.

Thus, when we use a percentage we express it by the corresponding fraction, i.e. r per cent. means $\frac{r}{100}$ths.

Example (1). *What is 7 per cent. of a ton weight?*

2240 lb. equal 1 ton.
$\therefore$ 2240 × $\frac{7}{100}$ is 7 per cent. of a ton in lb.
i.e. 156·8 lb.

Example (2). *In the operation of turning a job on a lathe,* $1\frac{1}{2}$ *per cent. of metal is waste; calculate the amount of waste on a job which weighs* 15 *cwt. before starting.*

15 cwt. = 15 × 112 lb. = 1680 lb.

$\therefore$ 1680 × $\frac{1\frac{1}{2}}{100}$ = **25·2 lb.**

Very commonly, percentage problems may be set out as ordinary proportion sums:

Example (3). *In* 30 *lb. of brass there are* 6 *lb. of zinc and the remainder is copper. Find the percentage amounts of zinc and copper in this brass.*

In 30 lb. of brass there are 6 lb. of zinc.
,, 1 ,, ,, ,, ,, is $\frac{6}{30}$ lb. of zinc.
,, 100 ,, ,, ,, ,, are $\frac{6}{30} \times 100$ lb. of zinc.
I.e. **20 per cent.** is zinc.
The remaining 80 per cent. is copper.

Example (4). *A circle is drawn in a square piece of sheet metal 20″ edge. Find the percentage waste when the circular disc is cut out of the square. The circle touches the edges of the square.*

The area of the square $= 20 \times 20 = 400$ sq. in.
,, ,, ,, ,, disc $= \frac{22}{28} \times 20^2 = \frac{22}{28} \times 400$ sq. in.

$$\text{Waste} = 400 - \frac{22}{28} \times 400 = 400\left(1 - \frac{22}{28}\right) = \overset{100}{\cancel{400}} \times \frac{6}{\underset{7}{\cancel{28}}} \text{ sq. in.}$$

Therefore, on 400 sq. in. the waste is $\frac{600}{7}$ sq. in.

,, 1 ,, ,, ,, ,, $\frac{600}{7 \times 400}$ sq. in.

,, 100 ,, ,, ,, ,, $\frac{600}{7 \times 400} \times 100$ sq. in.

$$\text{Cancelling : } \frac{\overset{150}{\cancel{600}} \times \cancel{100}}{7 \times \cancel{400}} = \mathbf{21\tfrac{3}{7}} \textbf{ per cent.}$$

Example (5). *A particular bronze contains by weight 21 parts copper and 4 parts tin. Calculate the percentage composition of the bronze.*

Whatever unit of weight is used :

In 25 parts, there are 4 parts tin.
,, 1 part there is $\frac{4}{25}$,, ,,
,, 100 parts there are $\frac{4}{25} \times 100$ parts tin.
$\frac{4}{25} \times 100 =$ **16% tin.**

The remaining **84 per cent. is copper.**

Example (6). *If 3 is to 5 is a ratio, what is its value per hundred or per cent.?*

Let x be the percentage—

$$\text{then } \frac{3}{5} = \frac{x}{100}. \quad \therefore x = \frac{3}{5} \times 100 = 60.$$

Hence $\frac{3}{5} = \frac{60}{100}$, i.e. **60 per cent.**

Example (7). *What per cent. is the ratio* $\frac{3}{12\frac{1}{2}}$?

Let x be the percentage—

$$\frac{x}{100} = \frac{3}{12\frac{1}{2}}. \quad \therefore x = \frac{3 \times 100}{12\frac{1}{2}}$$

$$x = 24,$$

i.e. 24 per hundred or **24 per cent.**

Example (8). *Fifteen per cent. increase on wages is given. What is the increase on a wage of £3 9s. per week?*

Let $x =$ the increase on 69*s*.

$$\frac{x}{69} = \frac{15}{100}$$

$$x = \frac{15}{100} \times 69 = \mathbf{10s.\ 4\tfrac{1}{4}d.}$$

Example (9). *A particular bronze is made of* 18 *parts weight copper and* 7 *parts tin. What percentage of each metal is in the bronze?*

The total number of parts is 25.

Let $x =$ the percentage of copper.

$$\therefore \frac{x}{100} = \frac{18}{25}. \quad \therefore x = \frac{18}{25} \times 100.$$

$$x = 72 \text{ per cent.}$$

Let $y =$ the percentage of tin.

$$\text{Then } \frac{y}{100} = \frac{7}{25}. \quad \therefore y = 100 \times \frac{7}{25}.$$

$$y = 28 \text{ per cent.}$$

Example (10). *How much of each metal is in one ton of the bronze in Example* (9)?

72 hundredths of the ton is copper.
28 ,, ,, ,, ,, ,, tin.

$$\therefore \text{copper} = \frac{72}{100} \times 20 \text{ cwt.} = 14\tfrac{2}{5} \text{ cwt.}$$

$$\text{tin} = \frac{28}{100} \times 20 \text{ cwt.} = 5\tfrac{3}{5} \text{ cwt.}$$

Example (11). *The agent for a company has a weekly allowance of £2 and a commission of* 2½ *per cent. on his sales. In one year his sales amounted to £8,000. Calculate his total income for the year.*

Allowance per year = 2 × 52 = £104.
2½ per cent. of his sales means 2½ hundredths of them.

$$\therefore £8{,}000 \times \frac{2\frac{1}{2}}{100} \text{ is his commission for the year.}$$

Cancelling 100 : £80 × 2½ = ,, ,, ,,
£200 = ,, ,, ,,
Hence, his total income is 200 + 104 = **£304.**

Example (12). *The efficiency of a machine is usually expressed as a percentage, from the expression*

$$\frac{x}{100} = \frac{\textit{Work got out of the machine}}{\textit{Work put into the machine}},$$ where x is the percentage efficiency.

$$\textit{I.e.}, \frac{\text{Work got out of the machine}}{\text{Work put into the machine to drive it}} \times 100 = \text{percentage efficiency.}$$

The brake horse-power of a machine measures the work got out, while the indicated power is the work put in. If the brake horse-power is 72 and the indicated horse-power is 90, what is the percentage efficiency?

$$\tfrac{72}{90} \times 100 = 80, \textit{ i.e.} \textbf{ 80 per cent.}$$

✓EXERCISE 8

(1) Out of 230 bolts made on a machine, 48 were termed scrap on account of defective threads. What percentage of the bolts was scrap?

(2) What is 6¼ per cent. of 144?

(3) If the cost of producing an article is increased from 6*s*. 3*d*. to 8*s*. 9*d*., find the percentage increase.

(4) A steel rod 66′ long stretches 3″ when under a direct pulling load. Find the percentage increase in length.

(5) The composition of a white metal was 48 parts copper, 14 parts tin, and 1 part antimony. What is the percentage of each metal?

(6) What is 9% of 660 tons of scrap?

(7) Plumber's solder contains 33⅓ per cent. of tin. How much tin is there in 84 lb. of solder?

(8) A particular iron ore yields 40% of iron. What amount of iron is yielded from 360 tons of ore?

(9) The coal consumed by a factory during a particular week was 5 per cent. less in 1943 than in the same week in 1942. If 150 tons of coal were consumed in the week of 1942, find the consumption in the week of 1943.

(10) A straight link of steel 8″ long and ⅝″ diameter was tested under a stretching load. It was found that just as the specimen broke the length was measured to be 10¼″ long and that the diameter at the break was reduced to ⅜″. Calculate the elongation per cent., and the reduction of area per cent.

(11) Some foundry sand was being tested for its porosity. A can of 800 cu. cm. capacity was rammed full of sand. After ramming, 240 cu. cm. of water were soaked into the rammed sand. Calculate the percentage of void, or porous space, there was in the can of sand.

✓(12) Two hundred and forty people work at a factory, three are away due to accidents, eight due to illness, and five are absent for other reasons. Find the percentage absenteeism in each case.

(13) A metal ore contained 15 per cent. of metal. If 75 per cent. of the refuse were removed by filtering, find the percentage metal in the remainder.

(14) In measuring a piece of work, one length was said to be 11·2″, whereas the correct length was 10·4″. What was the percentage error ?

(15) A metallic alloy was composed by weight of 1 part magnesium, 14 parts copper, and 48 parts aluminium. Find the percentage composition.

(16) In a certain copper ore 35 per cent. is copper. How much ore is required to yield 7 tons of copper ?

(17) The cost of producing an article was 8*s.* 3*d.* which included an additional 10 per cent. for carriage. What was the cost of producing the article without the carriage ?

(18) One hundred and twenty men each earned 1*s.* 8*d.* a day overtime. When the amount paid per man was increased by 20 per cent., it was found that overtime by 36 men could be dispensed with. Find the percentage decrease in the total money paid for overtime.

(19) In an experiment on the flue gases from a furnace flue, it was found that of the volume of gas tested 80 per cent. was nitrogen and that 60 per cent. of the remainder was carbon dioxide. After the above two gases had been removed, only 16 cu. in. of carbon monoxide remained. Calculate what volumes of nitrogen and carbon dioxide were present.

(20) A machine costing £100 depreciated 5% of its value during every year of its working life. What is the value of the machine at the end of the third year ?

(21) On a tensile test being made on a $\frac{5}{8}''$ diameter mild-steel rod 8″ long, it was found that it stretched 27 per cent. of its original length before it broke. What amount did it stretch ?

(22) Twenty thousand of a small job are produced per week by four machines. It is required that the production be increased by 16 per cent. How many extra articles per week on the average must each machine produce ?

(23) In making up a certain class of steel the following amounts of metals were fused together : $8\frac{2}{5}$ cwt. of iron, $1\frac{1}{5}$ cwt. of chromium, and $\frac{2}{5}$ cwt. of nickel. Calculate the percentage composition of the alloy.

CHAPTER IX

PLOTTING GRAPHS

A MOST striking way of conveying information respecting two related quantities is by making a pictorial view of the relation, and this is accomplished by plotting a graph.

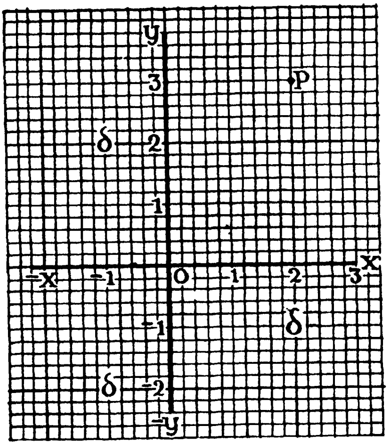


FIG. 28.

In order to plot a graph axes of reference are taken. These axes are two lines drawn at right angles to each other, as shown in Fig. 28. Their intersection, the point O, is called the **origin**, and values of the quantities are plotted in distances according to scale from these axes of reference. In Fig. 28 YOY is one axis and XOX is the other. In this graph, values of X and values of Y are plotted, though any other two letters may be used to represent the quantities to be plotted. Values of X along the horizontal axis XOX and values of Y along the vertical axis YOY.

Scales have to be set out along the axes of reference. In the above case $\frac{1}{2}''$ represents the value of one unit of either X or Y.

For the point P shown in the figure the value of X is 2 and the value of Y is 3.

For the present we will deal only with measurements along OX and OY.

When the student is about to construct a graph, the scales are very important. They should be taken of such amounts so that graphs as large as the paper allows are obtained.

The squared paper is usually divided into inches, half-inches, and tenths of an inch. This facilitates the plotting of graphs. In some cases, however, the squared paper has units of centimetres and millimetres, which makes it more convenient for plotting quantities on the centimetre, gram, system.

Example (1). *The following table contains results from an experiment on loading a spiral spring.* L *is the load on the end in pounds,* E *is the extension in inches.*

L lb.	0	2	4	6	8	10	12
E in.	·0	·25	·501	·75	1·01	1·251	1·5

The following graph (Fig. 29) illustrates the result of the experiments.

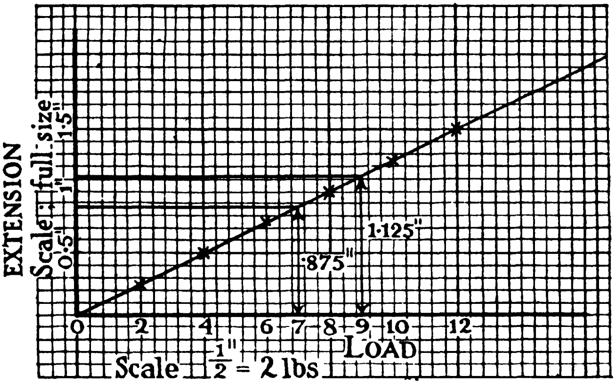


FIG. 29.

From this graph, within the range of values taken, the extension for any other load not mentioned in the table can be read. Thus for a load of 9 lb. the extension is 1·125". Similarly, the load to produce a given extension can be read. The load to produce an extension of 0·875" is 7 lb.

Example (2). *The following figures are taken from Croft's catalogue.* H = *the horse-power transmitted when the line shaft belt has a speed of* 1,000' *per minute.* W = *the width of belt in inches.*

W	2	3	4	5	6	7	8	9	10	11	12
H	3·56	5·24	7·12	8·9	10·68	12·46	14·24	16·02	17·8	19·58	21·36

Since the graph, Fig. 30, is a straight line, it shows that the horse-power transmitted is proportional to the width of the belt.

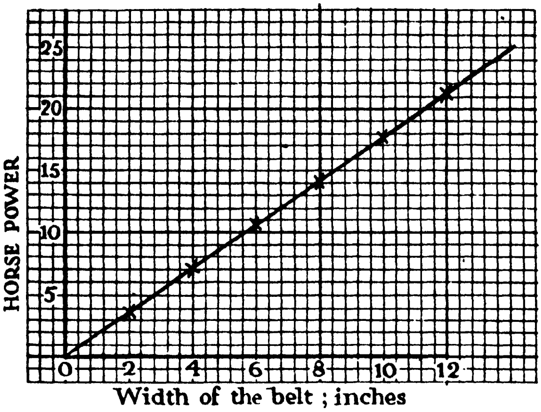


FIG. 30.

Example (3). *Several mild-steel bars of square section were tested for their strength under direct pulling loads. The following loads gradually applied were found to break the bars. Plot a graph.*

Edge of square in in.	¼	½	¾	1	1¼	1½	1¾	2
Breaking load in tons	1¾	7	15¾	28	43¾	63	85¾	112

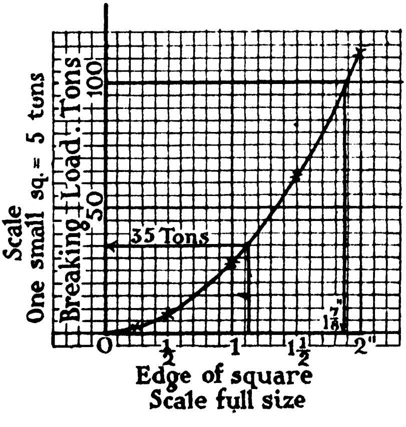


FIG. 31.

From the graph, read the breaking load for a 1⅛"-edge bar. What size of bar will break at 100 tons?

The load which will break a bar 1⅛" square in section is 35 tons.

The size of bar which will break under a load of 100 tons is 1⅞" square.

Example (4). *Construct a scale,* $\frac{1}{4}''$ *to one foot, to represent any amount up to* 12′.

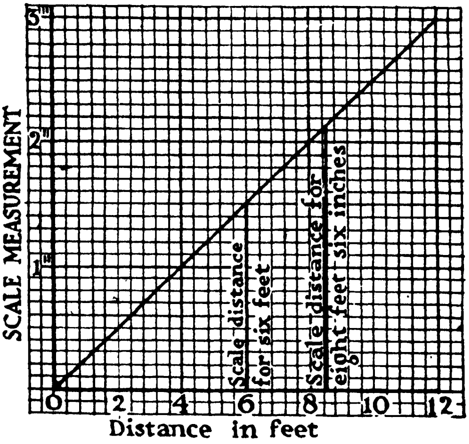


FIG. 32.

Measure off vertical height with dividers. This is the distance representing feet indicated at base, to required scale.

Example (5). *Construct a graph which may be used as a scale* 1″ *to* 20 *yards, from which any quantity from* 0 *to* 50 *yards can be measured.*

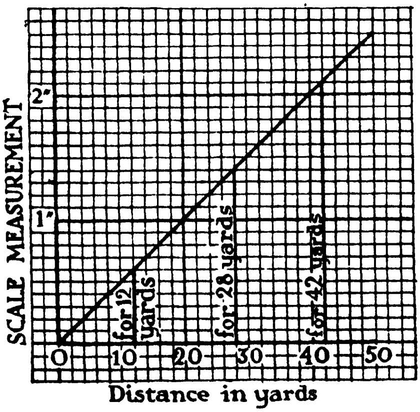


FIG. 33.

Example (6). *Plot the expression* $y = x^2$ *for values of* x *from* 1 *to* 12. *Let* x *have values* 1 *to* 12 *and get the corresponding values of* y.

$x =$	1	2	3	4	5	6	7	8	9	10	11	12
$y =$	1	4	9	16	25	36	49	64	81	100	121	144

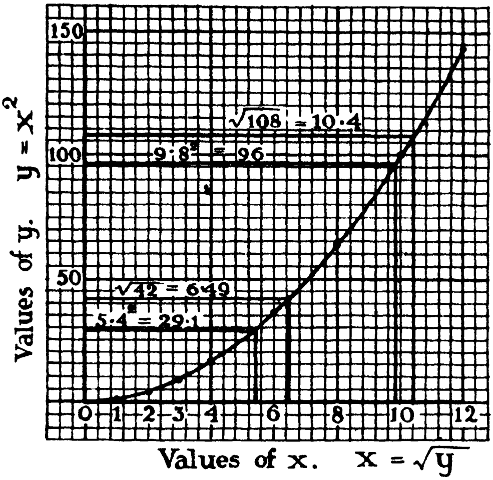


FIG. 34.

Read off value of $5{\cdot}4^2$, $9{\cdot}8^2$, $\sqrt{42}$, $\sqrt{108}$.

Example (7). *Plot the expression* $y = x^3$ *for values of* x 1 *to* 10.

x . .	1	2	3	4	5	6	7	8	9	10
y . .	1	8	27	64	125	216	343	512	729	1000

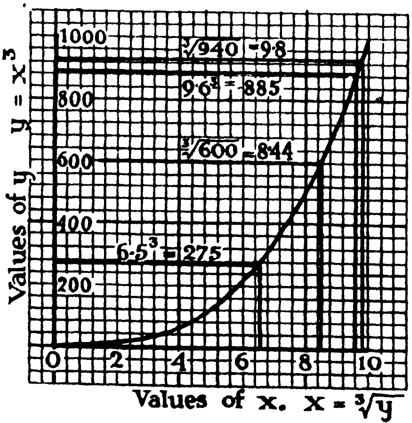


FIG. 35.

From the graph, read the values of:

$9{\cdot}6^3$, $6{\cdot}5^3$, $\sqrt[3]{940}$, and $\sqrt[3]{600}$.

✓EXERCISE 9

(1) The following figures are taken from a price list of adjustable spanners. S is the size and P is the price in shillings.

S, in.	½	¾	1	1¼	1½	1¾	2
P	5	8	10	12	15	19	21

Plot a graph illustrating price and size.

(2) V cubic feet is the volume of a certain weight of air at temperatures t° Fahrenheit. Plot a graph illustrating volume and temperature.

t°	54	66	80	110	136
V	110	112	115	120·1	125

(3) Plot a graph which may be used as a scale ¼″ to a foot from which may be taken any dimension to represent any number of feet from 0 to 40.

(4) Plot a graph which may be used as a scale 1½″ to a foot. Measure any dimension from it from 0 to 4 feet.

(5) H is the horse-power absorbed in driving a workshop and N is the number of machines employed or in use. Plot a graph of the figures given.

H	31	37	47	58	67	83	104	124	145	162
N	6	10	14	19	21	26	33	40	48	56

(6) The following figures are taken from Croft's tables, giving lengths in feet of steel shafting of various diameters per ton weight.

Diameter, inches	1	1½	2	2½	3	4	5	6	8
Length per ton	839	373	210	134	93	52	33½	23¼	10⅓

Plot a graph on diameter as base axis.

From the graph read off the lengths per ton of shafting having diameters: 2¼″, 3½″, 4¼″, 5¾″.

(7) The following table gives the breaking load of cotton driving ropes.

Diameter, inches	1	1¼	1½	1¾	2	2½
Breaking load, lb.	6400	9400	15000	22000	28500	40000

From the graph read the breaking loads for ropes having diameters: 1⅛", 1⅝", 2¼", 1⅞".

(8) Dealing with the pitches of teeth on tooth wheels, the following table gives the circular pitch inches and the diametral pitch. On a base of diametral pitch, plot a graph illustrating the table of figures.

Diametral pitch	·7854	·8976	1·5708	3·142	6·283	12·566
Circular pitch	4	3½	2	1	½	¼

(*a*) From the graph read off the circular pitch of wheels having a diametral pitch of 2½ and 1¼.

(*b*) Also read the diametral pitch of teeth having a circular pitch of 2¾" and 1¼".

(9) The following table gives the horse-powers that Croft's disc-type friction clutches will transmit at 1000 revolutions per minute.

H = horse-power. S = size of clutch.

S, inches	4½	6	8	12	14
H	6	16	38	130	210

From the graph read the horse-powers that clutches of diameters 5" and 10" will transmit.

(10) For keying pulleys to shafts the width W in inches of the key is obtained from the formula:

$$W = \frac{D}{4} + \frac{1}{8}'',$$

where D is the diameter of the shaft in inches.

Plot a graph illustrating width of the key on a base of diameters from 1" to 12".

(11) The figures below give the number of threads per inch N and the full diameter D of bolts having Whitworth screw threads:

D, inches	¼	⅜	½	¾	1	1½	2	2½	3
N	20	16	12	10	8	6	4·5	4	3·5

From the graph read the numbers of threads per inch for bolts having diameters $\frac{7}{16}''$, ⅞", 1¼", 1⅝", 2¾", respectively.

(12) The degree of temperature Centigrade is related to the temperature on the British or Fahrenheit scale by the formula :

$$C = \frac{5}{9}(F - 32).$$

Plot a graph on a base of temperature Fahrenheit from 60° to 300°, from which one can read C, the temperature Centigrade.

(13) The cost C pence each of articles produced by a firm depends upon the number produced P according to the table :

P	1000	1200	1400	1600	1800	2000
C	4	3·9	3·7	3·4	3·0	2·5

Taking the axis of P horizontally, illustrate these figures by means of a graph, choosing your own scale. By the aid of the graph quote a cost (per article) for an order of 1700 articles. What is the least number that could be produced for 3*s*. 6*d*. a dozen ?

(14) Plot a graph showing a scale $\frac{1}{2}''$ to 1 yard on which any measurement up to 20 yards is shown.

(15) Plot a scale $\frac{1}{8}''$ to a foot from which any measurement up to 40′ can be taken.

CHAPTER X

ALGEBRA

THE use and manipulation of " symbols," as explained in Chapter IV, is part of the subject called **Algebra.** To the engineer, the main use of **Algebra** is in the manipulation of formulæ. Usually, formulæ are built up of one or more letters, and it is in the arrangement and manipulation of these letters in order to find the value of the quantity required that a knowledge of algebra is of great importance.

At the outset, the student should know that although in algebra the use of letters is adopted, these letters stand for numbers, so that when the letters x and y are said to be multiplied together, we mean that the number corresponding to x is multiplied by the number corresponding to y. Similarly with other operations—addition, subtraction, etc., as are done in arithmetic. In the formula for the circumference of a circle, let C be circumference and d be diameter, then we know that the relation between C and d can be expressed in the formula :

$$C = \tfrac{22}{7}d, \text{ as stated on page 23.}$$

Or expressing π as a decimal fraction :

$$C = 3{\cdot}142d.$$

Given a value for d, C can be evaluated.

This formula is the simplest form of an algebraic equation, for wherever one algebraical expression can be expressed as equal to another the whole is called an equation. Thus all formulæ with the sign = between the two parts of them are equations. For example :

$$\text{Area of a circle of diameter } d \text{ is equal to } \frac{\pi d^2}{4}.$$

$$\text{If A = the area, then } A = \frac{\pi d^2}{4}.$$

When the value of d is given, A can be evaluated. Similarly, when the value of A is given, d can be found.

Sometimes one has to work so as to be correct to the fourth or fifth decimal place ; then the value of π should be taken as 3·1416 or 3·14159.

Before the student attempts to manipulate formulæ, the following six axioms or truths should be known perfectly :

(1) If equals be added to equals, the sums are equal.
(2) If equals be taken from equals, the remainders are equal.
(3) If equals be multiplied by the same number the results are equal.
(4) If equals be divided by the same number, the results are equal.

(5) The square root or cube root of one side of an equation is equal to the square root or cube root of the other side.

(6) The square or cube of one side of an equation is equal to the square or cube of the other side.

The Multiplication Sign

In Algebra the multiplication sign × **is usually** omitted. Thus, the student should read WLD as meaning the product of W, L, and D.

Thus in the formula :

$$H = \frac{\pi DNT}{33000},$$

it is understood that the values of π, D, N, and T are multiplied together and the result divided by 33000.

For instance, let $\pi = 3{\cdot}142$, $D = 3\frac{1}{2}$, $N = 220$, and $T = 90$. Then :

$$H = \frac{3{\cdot}142 \times 3\frac{1}{2} \times \overset{2}{\cancel{220}} \times \overset{3}{\cancel{90}}}{\underset{\underset{10}{\cancel{30}}}{\cancel{33000}}} = 6{\cdot}6.$$

Thus **H = 6·6.**

In the above calculation, H represents the horse-power transmitted by a belt pulley of diameter D feet revolving at N revolutions per minute when the effective force given by the pulley rim is T pounds.

Example (1). In the formula given above for the circumference of a circle :

$$C = 3{\cdot}14159d.$$

To obtain d in terms of C, divide both sides by 3·14159.

$$\text{Thus} \quad \frac{C}{3{\cdot}14159} = \frac{3{\cdot}14159d}{3{\cdot}14159}.$$

On the right-hand side, 3·14159 cancels and leaves the formula—

$$\frac{C}{3{\cdot}14159} = d.$$

Substituting the value of C, d may be evaluated.

Example (2). The volume of a sphere is given by the formula :

$$V = \frac{\pi d^3}{6},$$

where " V " is volume and " d " is the diameter.

Multiply both sides by 6 and divide both sides by π.

$$\text{Thus} \quad \frac{6V}{\pi} = \frac{6\pi d^3}{6\pi}.$$

Cancelling, we get—

$$\frac{6V}{\pi} = d^3.$$

Now take cube root of both sides :

$$\sqrt[3]{\frac{6V}{\pi}} = d.$$

In these two examples, the student will note that any of the operations done is according to one of the six axioms stated.

Example (3). *Find the value of x in the following equation—*

$$\frac{3}{x} = 4\tfrac{1}{2}.$$

$$\frac{3 \times \cancel{x}}{\cancel{x}} = 4\tfrac{1}{2} \times x, \text{ i.e. } 3 = 4\tfrac{1}{2} \times x.$$

$$\text{Again } \frac{3}{4\frac{1}{2}} = \frac{\cancel{4\frac{1}{2}} \times x}{\cancel{4\frac{1}{2}}}.$$

$$\text{This gives } x = \frac{3}{4\frac{1}{2}}$$

$$\text{or } x = \tfrac{2}{3}.$$

Example (4). *Find the value of y in the equation—*

$$6y - 12 = 2y + 18.$$

Add 12 to both sides, then $6y = 2y + 30$.
Take $2y$ from both sides, and $4y = 30$.
Divide both sides by 4, and $y = 7\tfrac{1}{2}$.

Example (5). *From the formula* $C = \frac{E}{R}$, *evaluate* R *when* $C = 12$ *and* E *is* 56.

Substituting the numbers for the letters—

$$12 = \frac{56}{R}.$$

Multiply both sides by R, i.e.

$$12R = \frac{56}{\cancel{R}} \times \cancel{R}.$$

$$\text{Thus } 12R = 56.$$

$$\text{This gives } R = \tfrac{56}{12} = 4\tfrac{2}{3}.$$

Example (6). *The coefficient of friction between two sliding surfaces is given by the formula—*

$$\frac{E}{P} = C,$$

where C *is the coefficient,* P *is the pressure between the surfaces, and* E *is the effort to cause sliding. If* E $= 15$ *and* C $= \cdot 02$, *find the value of* P.

$$\frac{15}{P} = \cdot 02.$$

Multiply both sides by P, and then

$$15 = \cdot 02P.$$

Divide both sides by ·02, and then—

$$\frac{15}{\cdot 02} = P,$$

$$\text{i.e.}\quad \frac{1500}{2} = P.$$

$$750 = P.$$

Example (7). *The percentage efficiency of a machine is given by the formula*—

$$e = \frac{M}{V} \times 100,$$

where e *is the percentage efficiency,* M *is the mechanical advantage, and* V *is the velocity ratio. If* $e = 45$, *and* $V = 20$, *find the value of* M.

$$45 = \frac{M}{20} \times 100.$$

Cancelling 20 into 100—

$$45 = M \times 5.$$

Divide both sides by 5, and we get $9 = M$.

Example (8). *The energy stored in a moving object of weight* W *lb. and with a velocity* V *feet per second is given by the formula*—

$$E = \frac{W \times V^2}{2 \times g},$$

where E *is the energy in foot-pounds and* $g = 32\cdot 2$. *If* $E = 25{,}000$, *and* $W = 560$ *lb., find the value of* V.

Substituting these values for the letters—

$$25000 = \frac{560 \times V^2}{2 \times 32\cdot 2}$$

Multiply both sides by 64·4, then—

$$25000 \times 64\cdot 4 = 560 \times V^2.$$

Divide both sides by 560, then—

$$\frac{25000 \times 64\cdot 4}{560} = V^2.$$

Take square root of both sides, then—

$$V = \sqrt{\frac{25000 \times 64\cdot4}{560}}.$$

$$V = 53\cdot62' \text{ per sec.}$$

Example (9). *If* $T = 2\pi\sqrt{\frac{L}{g}}$, *find the value of* L, *when* $T = 2$, $\pi = \frac{22}{7}$, *and* $g = 32\cdot2$.

Substituting the given values—

$$2 = 2 \times \tfrac{22}{7} \times \sqrt{\frac{L}{32\cdot2}}.$$

Multiply both sides by 7 and divide both sides by 2 × 22.

$$\frac{\cancel{2} \times 7}{\cancel{2} \times 22} = \sqrt{\frac{L}{32\cdot2}}.$$

$$\frac{7}{22} = \sqrt{\frac{L}{32\cdot2}}.$$

Square both sides :

$$\frac{49}{22 \times 22} = \frac{L}{32\cdot2}.$$

Multiply both sides by 32·2.

$$\therefore \frac{32\cdot2 \times 49}{22 \times 22} = L.$$

$$3\cdot26 = L.$$

✓ EXERCISE 10(a)

✓(1) A round steel bar has to have an area of 8 sq. in. Find the value of the diameter. The formula is : $\frac{3\cdot142d^2}{4} = 8$.

✓(2) In the formula $P = F + rs$, obtain the value of s when $P = 300$, $F = 25$, and $r = 15$.

✓(3) Find the value of R in the formula $C = \frac{E}{R}$, when $C = 15$ and $E = 400$.

✓(4) In the formula $2K + MU^2 = MV^2$, find the value of K, when $M = 500$, $U = 10$, and $V = 50$.

✓(5) From the formula $\frac{WV^2}{2g} = E$, find the value of E, when $W = 20$, $V = 45$, and $g = 32\cdot2$.

From the same formula find the value of W, when $E = 40{,}000$, $V = 30$, and g has the same value.

(6) Find the value of F from the formula $F = \frac{WV^2}{gr}$, when $W = 5$, $V = 60$, $g = 32{\cdot}2$, and $r = 1\frac{1}{2}$.

(7) If $Ft = \frac{1}{2}WV^2$, find the value of F, when $t = 5$, $W = 4$, and $V = 7{\cdot}07$.

(8) Find the value of x from the formula $x = \frac{4\pi^2}{T^2}$, when $\pi = 3{\cdot}142$ and $T = 1{\cdot}3$.

(9) If $S = ut + \frac{1}{2}at^2$, find the value of S when $u = 4{\cdot}4$, $t = 6$, and $a = {\cdot}338$.

(10) Find the value of T from the formula $T = 2\pi\sqrt{\frac{L}{g}}$, given that $\pi = 3{\cdot}142$, $L = 1{\cdot}29$, and $g = 32{\cdot}2$.

(11) The traverse of a lathe is $\frac{1}{16}''$ per revolution of the work and the lathe work makes 90 revolutions a minute. In what time will the tool traverse 2′ ?

(12) The traverse of a lathe tool is $\frac{1}{16}''$ and the feed is ·003″. If the work is 7″ diameter and makes 90 revolutions a minute, what amount of metal in cubic inches is turned off in 5 minutes ?

(13) The area of a circle must be $5\frac{1}{2}$ square inches. Calculate its diameter.

(14) Twenty nuts and bolts are found to weigh 4 lb. How many nuts and bolts are there in 1 cwt. ?

(15) A steel rod 21′ long must be saw-cut into 21 equal parts. If each cut cost $1\frac{3}{4}d.$ in labour, find the total cost of cutting.

(16) In the formula $I = \frac{\pi TD}{64{\cdot}4}(R^4 - r^4)$, find the value of I when $\pi = 3{\cdot}142$, $T = 1$, $D = 490$, $R = 12$, and $r = 10$.

(17) If $y = \sqrt{\frac{R^2 + r^2}{2}}$, find the value of y when $R = 12$ and $r = 10$.

(18) In the formula $I = \frac{w\pi r^4}{2g}$, find the value of I when $w = 490$, $\pi = 3{\cdot}142$, $g = 32{\cdot}2$, and $r = 8$.

(19) Evaluate y from the formula $y = \frac{LPV^2}{633000A}$, when $L = 736$, $P = 96$, $V = 2{\cdot}3$, and $A = 102$.

(20) The breaking weight W on a beam of rectangular section, b'' broad and d'' deep, is $\frac{kbd^2}{L}$, where k is a constant and L is the length of the beam. W is in tons. Find the breaking weight of a beam where $b = 3''$, $d = 4\frac{1}{2}''$, and L is 144″. Take k to be equal to 2·3.

(21) To convert temperatures Centigrade to Fahrenheit temperatures the following formula is used :

$$F = \tfrac{9}{5}\,C + 32.$$

(i) Find F when $C = 100$.

(ii) Mercury freezes at $-40°$ C. Find the equivalent value of F.

(iii) Nickel melts at $2645°$ F. Obtain its melting temperature C.

(iv) Liquid air boils at $-297°$ F. Find its boiling temperature C.

(22) A drop hammer falls from rest through $5\frac{1}{2}'$ before striking. At the instant of striking its velocity in feet per second is given by the formula $V^2 = 2 \times 32{\cdot}2 \times 5\frac{1}{2}$. Evaluate V.

(23) The inward pull exerted on an object weight W revolving in a circle of radius R at N revolutions a minute is $P = \frac{W}{g}\left(\frac{2\pi N}{60}\right)^2 R$. Find P, when $W = 3$ lb., $N = 150$, $\pi = \frac{22}{7}$, and $R = 1\frac{3}{4}$. $g = 32{\cdot}2$.

ALGEBRAIC MANIPULATIONS

In algebraic manipulations the signs plus "+" and minus "—" have the same meaning throughout as they have in ordinary arithmetic, the former meaning " add " and the latter " subtract " or take away.

Addition

Example (1). *Find the sum of* $2r + 3s + 4t$ *and* $5r - 2s - 3t$.

The best way is to put similar terms under one another, thus :

$$\begin{array}{r}
2r + 3s + 4t \\
5r - 2s - 3t \\
\hline
7r + s + t \\
\hline
\end{array}$$

Adding a minus quantity is equivalent to subtracting an equal positive amount.

Example (2). *Evaluate the sum of* :

$$5x - 3y + 2z,\ 7x + 8y - 4z.$$

$$\begin{array}{r}
5x - 3y + 2z \\
7x + 8y - 4z \\
\hline
12x + 5y - 2z \\
\hline
\end{array}$$

Example (3). *Find the sum of* :

$$3x^2 - x + 7,\ x^2 + 6x + 3,\ 2x^2 - 5x - 4.$$

$$\begin{array}{rrr}
3x^2 & - x & + 7 \\
x^2 & + 6x & + 3 \\
2x^2 & - 5x & - 4 \\
\hline
6x^2 & & + 6
\end{array}$$

Subtraction

Example (1). *Subtract* $3m + n + 5$ *from* $5m + 3n + 7$.

The best way is to change the signs in the bottom line and, then add.

$$\begin{array}{r}5m + 3n + 7\\ 3m + n + 5\\ 2m + 2n + 2\end{array}$$

Example (2). *Take away* $4x^2 - 7x - 5$ *from* $6x^2 + 3x + 4$.

$$\begin{array}{r}6x^2 + 3x + 4\\ 4x^2 - 7x - 5\\ \hline 2x^2 + 10x + 9\\ \hline\end{array}$$

Rule : To subtract a minus quantity, change the sign " − " to " + ", and then add.

Example (3). *Take* $5P^3 + 12s - 7$ *from* $3P^3 + 7s + 3$.

$$\begin{array}{r}3P^3 + 7s + 3\\ 5P^3 + 12s - 7\\ \hline -2P^3 - 5s + 10\\ \hline\end{array}$$

Multiplication

In ordinary arithmetic $10^2 = 10 \times 10$, and $10^3 = 10 \times 10 \times 10$, so in algebra $x^3 = x \times x \times x$, for x represents a number.

Also $a^5 = a \times a \times a \times a \times a$.

Similarly, $a^3 \times b$ may be written as a^3b or ba^3, for $a^3b = a \times a \times a \times b$ and $ba^3 = b \times a \times a \times a$, and these are the same in value.

If $a = 10$ and $b = 4$, then the product is 4×10^3 or 4000.

Example (1). *Multiply* a^2b *by* ab^3.

Writing these in full as above—

$$a^2b \times ab^3 = a \times a \times b \times a \times b \times b \times b = a^3b^4.$$

The rule is that **the indices of the same letter are added.**

Example (2). *Multiply* $a^3b + ab^2$ *by* ab.

Each of the terms a^3b and ab^2 is multiplied by ab; then the result is $a^3b \times ab + ab^2 \times ab$, *i.e.* $= a^4b^2 + a^2b^3$.

Example (3). *Multiply* $3x - 2y$ *by* 3.

This is $3x \times 3$ and $-2y \times 3 = 9x - 6y$.

Example (4). *Multiply* $3x^2 - 4y^3$ by $4xy$.

This $= 3x^2 \times 4xy - 4y^3 \times 4xy = 12x^3y - 16xy^4$.

By the rules of signs, a minus multiplied by a plus gives a minus result, ***and a minus quantity multiplied by a minus, gives a plus result.***

Long Multiplication

When the multiplier contains more than one term we proceed as shown below:

Example (5). *Multiply* $3a^2 + 2b - 3c$ *by* $2a + 5b$.

Multiply all the terms in the top row by each term in the bottom row and put similar terms in the same columns.

$$\begin{array}{llllll}
3a^2 + 2b - 3c & & & & & \\
2a + 5b & & & & & \\
\hline
6a^3 + 4ab - 6ac & & & & & \\
 & + 15a^2b + 10b^2 - 15bc & & & & \\
\hline
6a^3 + 4ab - 6ac & + 15a^2b + 10b^2 - 15bc & & & & \\
\hline
\end{array}$$

Example (6). *Multiply* $3m^2 - 2m + 2$ *by* $m^2 - m - n$

In this example the student will observe that a minus multiplied by a minus gives a plus result.

$$\begin{array}{rrrrr}
3m^2 & - 2m & + 2 & & \\
m^2 & - m & - 1 & & \\
\hline
3m^4 & - 2m^3 & + 2m^2 & & \\
 & - 3m^3 & + 2m^2 & - 2m & \\
 & & - 3m^2 & + 2m & - 2 \\
\hline
3m^4 & - 5m^3 & + m^2 & & - 2 \\
\hline
\end{array}$$

Division

If we wish to divide $5N^4$ by N^2 we proceed as follows:

Divide $5N^4$ by N^2.

Writing each in full, then—

$$\frac{5N^4}{N^2} = \frac{5 \times N \times N \times N \times N}{N \times N}.$$

Cancelling the Ns in the denominator with two of those in the numerator the result is $5N^2$.

Thus the rule is, *take the index of the divisor from the index of the numerator.*

Example (1). *Divide* $7x^5$ *by* $2x^3$.

This is $3\frac{1}{2}x^2$.

Example (2). *Divide* $5L^3 + 2L^2$ by $3L$.

$$\text{This is } \frac{5L^3}{3L} + \frac{2L^2}{3L}.$$

Dividing by $3L$ really means to divide by $3L^1$, but the index 1 is never written. Hence the division is $\frac{5}{3}L^2 + \frac{2}{3}L$.

Example (3). $6x^5 - 4x^3 + x^2 \div - 2x^2$.

$$\text{This is } \frac{6x^5}{-2x^2} - \frac{4x^3}{-2x^2} + \frac{x^2}{-2x^2}.$$

By the rule of signs a plus divided by a minus gives a minus, and a minus divided by a minus gives a plus. Therefore the result is: $-3x^3 + 2x - \frac{1}{2}$.

Brackets

Suppose that we have two lengths of wire, one x'' long and the other y''. Now suppose again that we want to write down in algebraic form twice the sum of their lengths, i.e. twice $x + y''$. If this be written $2 \times x + y$ it is not clear that the 2 is to multiply x only, or the sum $x + y$. To make this clear we enclose in " brackets " $x + y$, so as to show that the *whole* of this is to be multiplied. This we would write in the form $2(x + y)$. This means that the whole of $x + y$ is to be multiplied by 2.

If we wish to do without the brackets or " *remove* " them we should then write the product as $2x + 2y$.

The connection could be written thus:

$$2(x + y) = 2x + 2y.$$

Hence we get the rule that " *If an expression within brackets, is to be multiplied by some number, and we wish to remove the brackets, each term within the brackets must be multiplied by it.*"

Sometimes we wish to reverse all this, when we have a number of terms with a common factor.

Thus $3x + 3y$ could be written as $3(x + y)$.

Practical examples are given below:

Example (1). *Give a simple formula for the area of the annulus*, Fig. 35A.

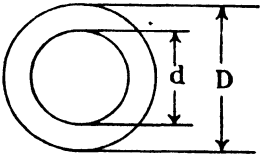


FIG. 35A.

$$\text{Area of the larger circle} = \frac{\pi}{4}D^2.$$

$$\text{Area of the smaller circle} = \frac{\pi}{4}d^2.$$

$$\text{Area of the annulus} = \frac{\pi}{4}D^2 - \frac{\pi}{4}d^2.$$

Taking the common factor $\frac{\pi}{4}$, and inserting brackets:

$$\text{Area of the annulus} = \frac{\pi}{4}(D^2 - d^2),$$

in which the brackets show that the whole of expression $D^2 - d^2$ is to be multiplied by $\frac{\pi}{4}$.

By inserting the brackets the evaluation is made easier.

Let $D = 6''$ and $d = 4''$.

$$\text{Then the area} = \frac{\pi}{4}(36 - 16) = \frac{3{\cdot}142 \times 20}{4}$$
$$= 3{\cdot}142 \times 5 = 15{\cdot}71 \text{ sq. in.}$$

Example (2). $3x(4y + 2z) = 12xy + 6xz.$
$$5m(2n - 7) = 10mn - 35m.$$
$$-2L(3x - 5) = -6Lx + 10L.$$

Multiplying each term inside the bracket by $-2L$, the signs of the terms inside the brackets are altered. Thus we get the rule: *When there is a minus factor in front of a bracket, "on removing the bracket, change the signs of all the terms in the bracket."*

Brackets within Brackets

If we find it necessary to use more than one pair of brackets, we use $\{\ \}$ or $[\ ]$.

Example.

Simplify $2\{3x + 5 - (2x - 7) + 6\}$.

The best method is to remove one bracket at a time, commencing with the inside bracket, thus:

$$2\{3x + 5 - 2x + 7 + 6\}$$
$$= 2\{x + 18\}$$
$$= 2x + 36.$$

In certain extreme cases of setting out, the student may use three sets of brackets, in this case a square bracket surrounds the whole.

Example. *Simplify*—

$$10[16 - \{3x - 2(4x + 7) + 8\} + 2]$$

Removing the innermost bracket first the expression

$$= 10[16 - \{3x - 8x - 14 + 8\} + 2]$$
$$= 10[16 - 3x + 8x + 14 - 8 + 2]$$
$$= 240 + 50x.$$

Some writers of text-books and also contributors to engineering journals use the hyphen as a bracket, thus—

$\overline{3P - 5}$ takes the place of $3(P - 5)$,

the hyphen being drawn over the top of the quantity.

Example. *Simplify*—

$$\frac{n}{2}(2a + \overline{n - 1}d)$$
$$= \frac{n}{2}(2a + nd - d)$$
$$= na + \frac{n^2d}{2} - \frac{nd}{2}$$

EXERCISE 10(b)

Multiply :

(1) $3x + 2y$ by 4.
(2) $5m - 3n$ by 6.
(3) $3L^2 - 2L$ by L.
(4) $5b^3 + 3b^2 - 2b$ by b^2.
(5) $6x^3 + 4xy - y^3$ by $2xy$.
(6) $3x - 2y$ by $3x + 2y$.
(7) $x^2 - xy + y^2$ by $x + y$.
(8) $x^2 + xy + y^2$ by $x - y$.

Find the sum of :

(9) $3a - 2b + c$ and $2a + 2b - c$.
(10) $7m + 3n + P$ and $- 5m - 4n - 2P$.
(11) $3a^2b + 5a - 7$ and $- 2a^2b + 2a + 3$.

Take away :

(12) $3a + 5P$ from $6a + 7P + 4$.
(13) $5a^2b + 4s - 2$ from $8a^2b + 7s - 3$.
(14) $8m^2n^2 + 2mn + 6$ from $3m^2n^2 + 4mn + 2$.

Divide :

(15) $14C^3$ by $7C$.
(16) $4x^4 - 3x^3$ by x^2.
(17) $30N^3 - 15N^2$ by N^2.
(18) $24d^3 - 3d^2$ by $3d$.
(19) $40P^3 - P^5$ by $- P^3$.
(20) $16R^2 - 20R^4$ by $- 4R^2$.

Simplify :

(21) $3(m + n) - 5(m - n)$.
(22) $a(a + b) + x(x - y)$.
(23) $5x - \{3y - 2(x - y)\}$.
(24) $6a + \{5a - (4a + 7)\}$.
(25) $\{5m + (3m + \overline{7 - m})\}$.

EQUATIONS

It has previously been stated that when one algebraical expression is equal to another, an equation is formed.

The simplest form of equation is of the type $4x = 24$, from which the student will easily observe that the value of x is 6.

Equations are said to be *solved* when the value of the unknown quantity is thus found, and the process is called the solution of the equation.

If the equation were expressed as $0{\cdot}4x = 24$, it requires a trifle more appreciation of the laws which govern the solution of equations. The student should revise and learn thoroughly the six axioms or self-evident truths laid down on page 56, for in the great majority of cases these axioms will be used in the solution of any equations which have to be solved.

Restating the latter equation :

$$0{\cdot}4x = 24.$$

Divide both sides by 0·4, and thus—

$$x = \frac{24}{0{\cdot}4} = 60.$$

Example (1). *Find the value of* P *from the equation*—

$$3P - 2 = P + 5.$$

Subtract one P from both sides.

$$\text{Thus } 3P - P - 2 = 5.$$

The student will observe that this is equivalent to bringing the P from the right-hand side and altering the sign from plus to minus.

The next step is, add 2 to both sides.

$$\text{Thus } 3P - P = 5 + 2.$$

This is equivalent to taking the − 2 from the left-hand side to the right and altering the sign from minus to plus.

The student will adopt this rule: *Collect the letters on one side and the figures on the other by transferring quantities from one side of the equation to the other and altering the sign to plus or minus as the case requires.* He must recognise that there is no mystery about the operation and that he is just adding or subtracting the same quantity to both sides according to the aforesaid axioms, page 56.

Example (2). *Find the value of* r *from the equation*—

$$0{\cdot}5r + 2r - 4 = r + 8.$$

Collecting the r's on one side and the figures on the other:

$$2{\cdot}5r - r = 8 + 4.$$

$$1{\cdot}5r = 12. \quad \therefore r = \frac{12}{1{\cdot}5}.$$

$$r = 8.$$

Example (3). *Solve the equation*—

$$2{\cdot}5 - (3v + 7) = - 5v + 30.$$

Removing the bracket—

$$\begin{aligned} 2{\cdot}5 - 3v - 7 &= - 5v + 30 \\ 5v - 3v &= 30 + 7 - 2{\cdot}5 \\ 2v &= 34{\cdot}5 \\ v &= 17{\cdot}25. \end{aligned}$$

The student should solve the equations in the following exercises as drill in the manipulation of the algebraic symbols.

✓EXERCISE 10(c)

Solve:

(1) $5y - 40 = 0$. (2) $5y + 20 = 0$. (3) $4(v - 2) = 3(v + 1)$.

(4) $2\frac{1}{2}(M - 4) = 2(M + 1) + 6$. ✓(5) $1{\cdot}8L - 9 + 2L = 19$.

Further Examples

The following examples introduce another operation.

Example (1). *Solve the equation—*

$$\frac{(2P + 1)}{10} = \frac{(2P - 1)}{15}.$$

In this type of equation involving denominators to fractions, the best method is to multiply each term by the lowest common denominator ; this is in accordance with one of the axioms stated and the operation will clear the equation of all denominators.

The lowest common denominator is 30 :

$$\therefore \quad \frac{30(2P + 1)}{10} = \frac{30(2P - 1)}{15},$$

$$i.e. \quad 3(2P + 1) = 2(2P - 1).$$

Removing brackets :

$$6P + 3 = 4P - 2.$$

$$2P = -5.$$

$$\therefore P = -2\tfrac{1}{2}.$$

Example (2). $\frac{3x}{35} + \frac{2}{5} = \frac{x}{5} - \frac{6}{7}.$

Multiply through by 35.

$$\therefore \quad 3x + 7 \times 2 = 7x - 5 \times 6.$$

$$3x + 14 = 7x - 30.$$

$$44 = 4x. \quad \therefore x = 11.$$

✓**EXERCISE 10(d)**

Solve the equations :

✓(1) $\frac{y - 2}{35} = \frac{y - 5}{14}.$

(2) $\frac{V}{11} + \frac{13}{55} = \frac{V}{5} - \frac{1}{11}.$

(3) $\frac{5}{7} + \frac{b - 2}{2} = \frac{3b}{7} + \frac{1}{14}.$

(4) $\frac{m}{3} - \frac{13}{30} = \frac{1}{5} + \frac{5m - 13}{10}.$

Equations in which the letter is in the denominator

Example (1). $\frac{20}{x} = 2{\cdot}4.$ *Find the value of x.*

Multiply each side by x :

$$\therefore \quad x \times \frac{20}{x} = 2{\cdot}4x.$$

On the left-hand side, x cancels out.

$$\therefore 20 = 2{\cdot}4x.$$

The student should observe that when we are said to multiply by x, we are really multiplying by the number corresponding to x, for it has been stated previously that in all formulæ or equations the letters stand for numbers.

$$\text{Since}\quad 2{\cdot}4x = 20,\ \text{then}\ x = \frac{20}{2{\cdot}4}.$$

$$\therefore\quad x = 8{\cdot}33.$$

Example (2). *Solve the equation—*

$$C = \frac{E}{R},\ \textit{where}\ C = 5\ \textit{and}\ E = 300.$$

Substituting these numbers:

$$5 = \frac{300}{R}.\quad \therefore 5R = 300.$$

$$\text{And } R = 60.$$

Example (3). *Solve the equation—*

$$\frac{5}{V} = 10 - \frac{7}{V}$$

Multiplying through by V—

$$5 = 10V - 7$$
$$12 = 10V$$
$$1{\cdot}2 = V.$$

Example (4). *Solve the equation—*

$$\frac{3}{5} - \frac{8}{x} = \frac{1}{3}.$$

The lowest common denominator is $15x$. Multiplying through by this—

$$9x - 120 = 5x$$
$$\therefore 4x = 120$$
$$\text{And } \therefore\ x = 30.$$

Example (5). *Solve the equation—*

$$\frac{44}{(2x+6)} = \frac{18}{x}$$

Multiply through by $x(2x + 6)$.

$$44x = 18(2x + 6)$$
$$44x = 36x + 108$$
$$8x = 108$$
$$x = 13\tfrac{1}{2}.$$

✓EXERCISE 10(e)

Solve the equations:

(1) $\frac{10}{V} = \frac{4}{V} + 3.$ (2) $\frac{9}{r} = \frac{3}{5}.$ (3) $\frac{25}{L} = \frac{35}{L} - 40.$

(4) $\frac{12}{(x+2)} = \frac{3}{x}.$ (5) $\frac{2 \cdot 5}{x} = \frac{7 \cdot 5}{x} - 30.$

(6) What is the value of the letter in each of the equations?

(i) $\frac{M}{50} = \frac{3}{4}.$ (ii) $\frac{N}{72} = \frac{7 \cdot 5}{9}.$ (iii) $\frac{P}{2 - 1\frac{1}{5}} = \frac{3}{18}.$

(7) If $H = \frac{P, L, A, N}{33000}$, state P, L, A, and N, as the subject of the formula in turn.

If $P = 100$, $L = 3$, $A = 85$, and $N = 440$, find the value of H.

(8) The area of a triangle is given by the formula—

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where A is the area, a, b, and c are the lengths of the sides, and

$$s = \frac{a+b+c}{2}.$$

Find A: (i) when $a = 24''$, $b = 26''$, and $c = 30''$.
(ii) when $a = 6''$, $b = 6\frac{1}{2}''$, and $c = 7\frac{1}{2}''$.

(9) Find the value of the letter in the equations:

(i) $4 \cdot 2A - 5 \cdot 1 = 2 \cdot 4A + 2 \cdot 1.$

(ii) $\frac{x}{2} + \frac{7}{8} + \frac{x-2}{5} = x + \frac{3 \cdot 5}{20}.$

(iii) $2 \cdot 4(2b - 3) = -5 \cdot 6(6b + 5).$

(10) Solve the equation:

$$7\frac{1}{2}N + 3\frac{1}{3}N = 10N + 3\frac{1}{3}.$$

✓(11) Find the value of x in the equation:

$$\frac{4(x-6)}{3} = 7 - \frac{5(x-4)}{2}.$$

PROBLEMS ON EQUATIONS

Example (1). *A line 35″ long is divided into two parts, one of which is two-thirds of the other in length. What are the lengths of the two parts?*

Let x'' be the length of the longer part. Then $\frac{2}{3}x$ is the length of the shorter part.

$$\therefore \quad x + \tfrac{2}{3}x = 35$$
$$1\tfrac{2}{3}x = 35$$

$$\therefore \quad x = \frac{35}{1\frac{2}{3}} = \frac{35}{\frac{5}{3}} = \frac{35 \times 3}{5}.$$

$$\text{Hence } x = 21''.$$

The shorter part is $\frac{2}{3} \times 21 = 14''$.

Example (2). *A particular brass is to be made, such that there are two parts by weight of zinc to one of copper. How much of each metal must be used to make* 10 *tons of this brass?*

Let x ton be the amount of copper. Then $2x$ tons is the amount of zinc.

$$\therefore \quad x + 2x = 10$$
$$3x = 10$$
$$x = 3\tfrac{1}{3} \text{ tons.}$$

Amount of copper $= 3\frac{1}{3}$ tons.
Amount of zinc $= 2x = 6\frac{2}{3}$ **tons.**

Example (3). *The perimeter of a rectangle is* 36" *and the breadth is* 3" *shorter than the length. Calculate the length and breadth.*

Let x'' be the length. Then $(x - 3)''$ is the breadth.

$$\text{Hence } 2x + 2(x - 3) = 36.$$

Divide through by 2, hence $x + (x - 3) = 18$.

$$i.e.\ 2x - 3 = 18, \therefore 2x = 21.$$
$$x = 10\tfrac{1}{2}''.$$
$$x - 3 = 7\tfrac{1}{2}''.$$

Hence the **length is** $10\frac{1}{2}''$ and the **breadth** $7\frac{1}{2}''$.

Example (4). *The charge for a foundry cupola is a quantity by weight of pig iron,* $\frac{1}{5}$ *of its weight of coke, and* $\frac{1}{20}$ *of its weight of limestone. If the whole charge is* 35 *cwt., find the weight of each constituent.*

Let x cwt. be the amount of pig iron.
Then $\frac{x}{5}$,, is ,, ,, ,, coke,
and $\frac{x}{20}$,, ,, ,, ,, ,, limestone.

Then $x + \frac{x}{5} + \frac{x}{20} = 35$.

$$1\tfrac{1}{4}x = 35, \quad x = \frac{35}{1\frac{1}{4}} = \frac{35}{\frac{5}{4}} = 35 \times \tfrac{4}{5}.$$

$$\therefore x = 28 \text{ cwt.} = \text{weight of pig iron.}$$

$$\frac{x}{5} = 5\tfrac{3}{5} \text{ cwt.} = \text{weight of coke.}$$

$$\frac{x}{20} = 1\tfrac{2}{5} \text{ cwt.} = \text{weight of limestone.}$$

EXERCISE 10(f)

(1) From the expression $39 = \sqrt{15^2 + b^2}$, find the value of " b."

(2) Find the value of y in the equation $\frac{50}{y} = \frac{48}{36}$.

(3) From the equation $\frac{72}{z} = \frac{9}{7\frac{1}{2}}$, find the value of z.

(4) In the equation $\frac{4}{m} = \frac{6}{5}$, what is the value of m?

(5) A pulley is x' diameter and makes N revolutions a minute. If the pulley drives a belt, give a formula for the speed of the belt: (*a*) in feet per minute, and (*b*) in feet per second, and (*c*) miles per hour.

(6) In the formula $\frac{\sqrt{ab}}{a - b} = \frac{2}{3}$, what is the value of a when $b = 3$?

(7) From the expression $a = \pi r^2$, find the value of r when $a = 154$.

(8) The perimeter of a rectangular piece of metal must be 114″. The length must be 3″ longer than the breadth. What are the dimensions?

(9) The distance fallen by a freely falling object is given by the formula:

$$d = 16t^2,$$

where d is the distance fallen in feet and t is the time in seconds. A chimney shaft is 256′ high, how long would it take an object to fall from top to bottom?

(10) How long would it take a drop hammer to fall 4′, the motion being according to the same law as the last example?

(11) In the formula $V = \frac{4}{3}\pi r^3$, what is the value of r when $V = 179\frac{2}{3}$? $\pi = 3\frac{1}{7}$.

(12) From the formula $F = \frac{9C}{5} + 32$, what is the value of C when F is 50?

Also when $F = -40$?

(13) Find the value of d in the formula $t = \sqrt{\frac{2d}{g}}$, when $t = 15$ and $g = 32$.

(14) Solve the equation $2{\cdot}4(2y - 3) + 5{\cdot}6(6y + 5) = 0$.

(15) A certain lathe costs four times as much as a particular drilling machine. If the cost of two lathes and five drilling machines was a total bill of £650, what was the price of each machine?

(16) In the equation $34{\cdot}05 = 34(1 + {\cdot}0000105t)$, calculate the value of t.

(17) A number of bags of bolts make up the weight of 10 tons. There are a number of half-hundredweight bags and twice as many hundredweight bags. How many of each were there?

(18) A line 37″ long is to be divided into three parts such that the second part is 2″ longer than the first and the third part is 3″ longer than the second. What are the lengths of the three parts ?

(19) The cross-section of a round steel rod has to be 7·07 sq. in. in area. Calculate its diameter.

(20) The perimeter of a rectangular area must be 20″ and the length must be 2″ longer than the breadth. What dimensions are the length and breadth ?

(21) From a metal rod L feet long, n pieces are cut off, each 6″ long. Allow $\frac{1}{8}$″ for each saw-cut and state the amount left on the rod in feet as a formula.

If $L = 12'$ and $n = 8$, what length is left ?

(22) Solve the equations :

(*a*) $\frac{M}{5} + \frac{16}{5} = \frac{2M}{3} - 1.$ (*b*) $\frac{a+3}{4} - \frac{a-3}{5} = \frac{a-5}{2} - 2.$

(23) (*a*) If $2{\cdot}6N - 3{\cdot}5 = N + 7{\cdot}7$, calculate the value of N.

(*b*) $3{\cdot}142 \times R^2 = 12{\cdot}568$, find the value of R.

(24) In the following equations, find the value of the letter :

(*a*) $3{\cdot}4L - 8 = 2({\cdot}6L + 7).$

(*b*) $V + \frac{7V+1}{5} = \frac{V-1}{5} + 2V.$

(25) In an experiment to test the friction of a planing-machine table it was found that a load of 1000 lb. was just moved by an effort of 15 lb. and a load of 500 lb. by an effort of 9 lb. Find a formula connecting force (F lb.) with the load (W lb.) assuming that it is of the type $F = aW + b$. Hence, find the force required to just move 1200 lb.

(26) A brass has to be made from melting copper and zinc together. Copper costs £135 a ton and zinc £60 a ton. The combined metal of which the brass is made must cost £108 a ton. Find the amount of copper and the amount of zinc by weight in one ton of brass.

(27) (*a*) A gear-wheel problem gives rise to the equation $D = \frac{N}{P \cos \theta}$.

If $D = 4{\cdot}83$, $N = 22$, and $P = 4{\cdot}75$, calculate the value of the angle θ.

(*b*) If $M = 82{\cdot}4 + \frac{10{\cdot}9}{L}$, calculate the value of L when $M = 86{\cdot}8$.

CHAPTER XI

ANGLES AND ANGULAR MEASUREMENT

When two lines meet at a point, they are said to form an angle. An angle is also formed when one line rotates about a fixed line around a point in it. From this latter definition it can be understood that a crank revolving about its centre makes various angles with the dead centre line of an engine or other machine, during a revolution.

Unit of Angular Measurement

One method of measuring angles is that in which the unit of angular measurement is called a " degree."

An angle of one **degree** is formed when a line rotates, as stated above, through $\frac{1}{360}$ of a complete revolution.

In Fig. 36, OA represents a **crank** which rotates, and A_1, A_2, A_3 are successive positions of A as the crank rotates, showing the angles between the crank and the centre line.

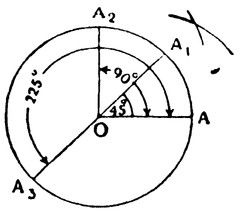


Fig. 36.

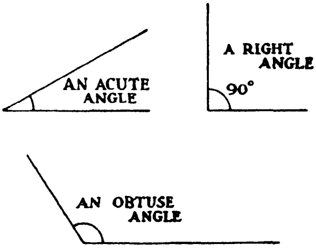


Fig. 36a.

In the above sketches of angles, Figs. 36 and 36(*a*), it is observed that a **right angle**, being just a quarter of a complete revolution, contains 90°. An **acute angle** is less than 90°, while an **obtuse angle** is greater than 90°.

Measurement of Angles

The Protractor

The protractor is an instrument by which angles are set out.

Fig. 37 represents a protractor, the angle between the radii being 10°. In the ordinary instrument each interval is divided into ten equal parts each representing one degree.

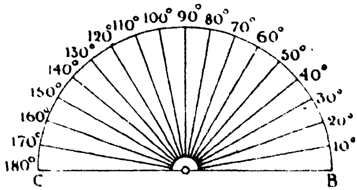


Fig. 37.

Example. Set out an angle of 40° from a point A in a line. Place the point O on the point A, and the line CB on the protractor along the given line. Make a dot opposite the line from O to 40°. Join this dot to the point A. An angle of 40° will be set out.

Triangles

A triangle is a plane figure enclosed by three straight lines.

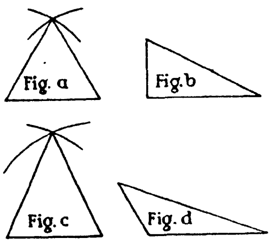


FIG. 38.

Fig. 38 (*a*) is an **equilateral triangle** as *all the three sides are equal in length*, and the three angles are equal to one another.

Fig. 38 (*b*) shows a **right-angled triangle,** *one of the angles being* 90°. The side opposite the right angle is called the hypotenuse.

Fig. 38 (*c*) is an **isosceles triangle,** *two of the sides are equal*, and the two angles opposite these equal sides are equal.

Fig. 38 (*d*) shows an **obtuse-angled triangle** —one angle being greater than 90°.

Fig. 38 (*a*) and (*c*) are also **acute-angled triangles,** *because each of its angles is less than* 90°.

Construction of Triangles

Example (1). *Given the lengths of the three sides*—3", 3½", 4".

Draw AB 4" long (Fig. 39). With centre A and radius 3½", construct an arc. With centre B and radius 3", construct another arc. At the point C where the two arcs intersect, join lines CA and CB.

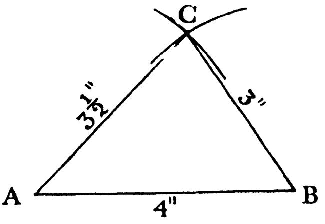


FIG. 39.

Example (2). *Given the lengths of two sides and the angle between them*—2½", 2", 50°.

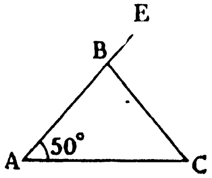


FIG. 40.

Draw AC 2½" long (Fig. 40). Draw line AE at an angle of 50° with AC. Mark off 2" along AE, *i.e.* AB. Join CB.

Example (3). *Given the length of one side and the two adjacent angles*—3″, 40°, 60°.

Mark off DE 3″ along the base (Fig. 41). At D draw a line at an angle 40° with DE. At E draw a line making an angle of 60° with ED. F, where the two lines intersect, is the vertex of the triangle.

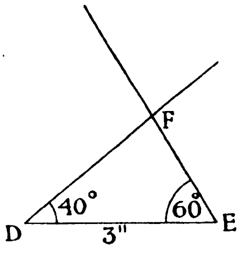


FIG. 41.

Example (4). *Given two sides of a triangle and the angle opposite one of the sides*—2″, 2½″, angle opposite side 2½″ = 45°.

Draw a line MN (Fig. 42). Set off a line at 45° with MN from M. Mark off MP = 2″. With centre P and radius 2½″, describe an arc cutting MN produced in Q.

The figure MPQ is the triangle required.

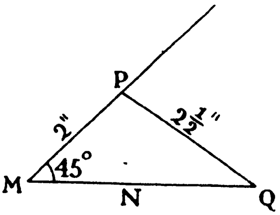


FIG. 42.

Ambiguous Case

The information given is the same as in the last example, except that the angle given is opposite the shorter side. Sides of lengths 1½″, 2″, and an angle of 30° opposite the 1½″ side, are the given quantities.

Draw a line PQ (Fig. 43). Set off from P a line making an angle 30° with PQ. Measure PO = 2″. With centre O and radius 1½″ describe an arc cutting PQ in R and S. Join OR and OS.

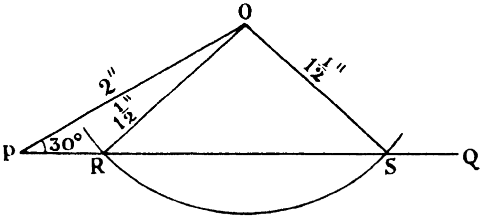


FIG. 43.

Thus, **two triangles may be constructed,** both of them satisfying the conditions. This is called the **Ambiguous Case,** in the construction of triangles.

It should be noted that this case arises when *the given angle is opposite the shorter side.*

Angles Less than One Degree

There are smaller units of angular measurement than the degree. One degree is divided into 60 equal parts called minutes, and each minute is divided into 60 equal parts called seconds. Seconds are never used in engineering, but the minute is commonly used.

Cutting Angles of Tools

The angles at the cutting edge of a tool are usually denoted as: (1) " Top Rake," (2) the " Cutting Angle," and (3) " Clearance." There is also " Side Rake " on a tool used for cutting parallel to the line between centres on a lathe.

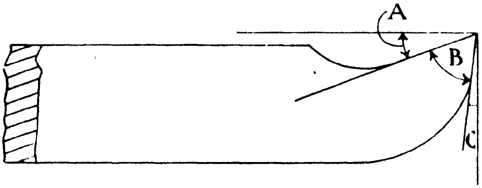


FIG. 44.

In Fig. 44, A is the top rake, B is the cutting angle, and C is the clearance.

The angles of cutting tools vary with the type of metal being worked upon. Tough metal tends to gather on the tool face and tear the surface. To resist this defect, a good top rake is allowed.

TABLE OF CUTTING ANGLES FOR STEEL AND BRASS

Metal.	*Top Rake.*	*Cutting Angle.*	*Clearance.*
		Degrees.	
Mild steel	37 to 22	50 to 60	3 to 8
Medium hard steel . .	33 to 19	54 to 63	do.
Hard steel	22 to 14	65 to 78	do.
Soft brass	24 to 10	63 to 74	do.
Medium brass . . .	17 to 7	70 to 75	do.
Hard brass	17 to 6	60 to 80	do.

In modern **very high-speed** metal cutting, a **negative rake** is allowed at the **tip** of the cutting tool. That is, instead of angle A, Fig. 44, being declined below the horizontal line, the top surface of the tool tip is elevated above the horizontal up to about 5 or 6 degrees.

Sum of the Angles of a Triangle

The sum of the three internal angles of any triangle is equal to two right angles or 180°.

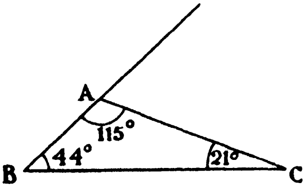


FIG. 45.

In the triangle ABC (Fig. 45), the internal angles should be measured. If this is done accurately, it will be found that the sum of them is 180°.

Similarly, in any other triangle, the sum of the three angles will be found to equal 180°.

Parallel Lines

Lines which are always the same distance apart are said to be parallel. They will never meet, no matter how far they may be produced. A line cutting through parallel lines is commonly called a **transversal.**

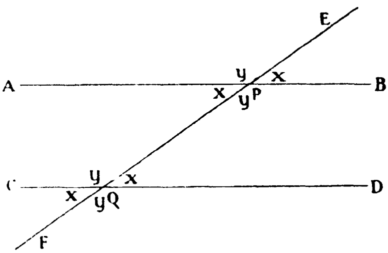


FIG. 46.

The lines AB and CD (Fig. 46) are parallel, and the line EF is a transversal. The angles made by a transversal and parallel lines are important.

Vertically opposite angles marked x and y are equal. Angle EPB is vertically opposite to angle FPA. These angles are equal to each other. Angle EQD is vertically opposite to angle FQC and they are therefore equal to each other.

Alternate angles are equal to each other. Thus the angle APF is alternate to angle EQD—these angles are equal to each other.

Hence the angles marked with an x are all equal to one another. Similarly all the angles marked with a y are equal to one another.

Angle BPQ plus angle PQD = 180°.

Example (1). In the figure ABCD (Fig. 47), AB and DC are parallel. Find angles A and B.

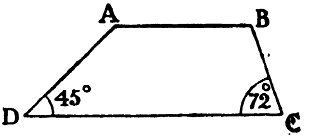


FIG. 47.

Angle A + angle D = 180° (as stated above).

∴ Angle A = 180° − 45° = 135°.

Similarly, angle B = 180° − 72° = 108°.

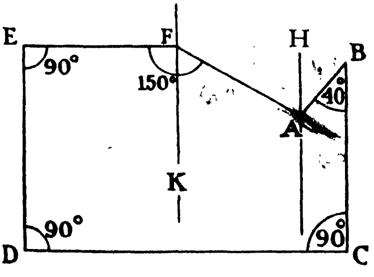


FIG. 48.

Example (2). Set out Fig. 48 to scale.

Measure angle A.

Then calculate angle A.

Set up lines AH and FK parallel to BC and DE at point A and at point F.

Angle EFK = 90°, ∴ angle KFA = 60°.

Since lines FK and AH are parallel and line FA is a transversal, then *the alternate angles* KFA *and* FAH *are equal*; that is, angle FAH = 60°.

Also, angle HAB is alternate to angle at B. ∴ angle HAB = 40°.

Hence the whole **angle at A = 100°.**

Divide a Straight Line into a Number of Equal Parts

Example. *Divide the line* AB *into ten equal parts* (Fig. 49).

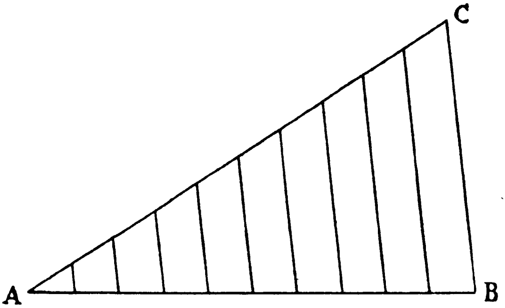


FIG. 49.

Draw AC at some convenient angle to AB and ten units long. Join C to B. From each unit along AC, draw a line parallel to CB to cut AB. The line AB will now be divided into ten equal parts.

The Exterior Angle of any Triangle is Equal to the Sum of the Two Interior Opposite Angles

In Fig. 50, BC, a side of the triangle ABC, is produced to D. ACD is an exterior angle.

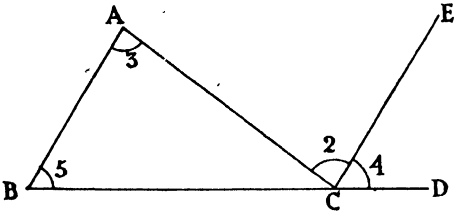


FIG. 50.

To show that the sum of the inside angles at A and B is equal to the exterior angle ACD.

Proof.—Draw CE parallel to BA.

Then, since BA and CE are parallel and CA cuts through them, angle 2 equals angle 3, being alternate angles.

Also, angle 4 equals angle 5, *i.e.* the exterior angle 4 is equal to the opposite interior angle 5. Hence the sum of angles 2 and 4 is equal to the sum of the interior angles 3 and 5.

The Sum of the Internal Angles of any Figure bounded by Four Straight Lines, *i.e.* a Quadrilateral, equals 360°.

Let ABCD be any four-sided figure (Fig. 51). Draw the diagonal DB and divide the figure into two triangles, ADB and CDB.

The sum of the angles 1, 2, and 3 = 180°.
The sum of the angles 4, 5, and 6 = 180°.

Hence the total sum of the four internal angles = 360°.

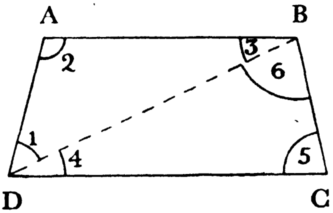


FIG. 51.

Parallelograms

A parallelogram is a four-sided figure whose opposite sides are equal and parallel (Fig. 52).

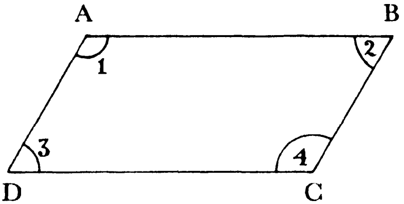


FIG. 52.

In the parallelogram ABCD, AD and BC are equal and parallel Also AB and DC are equal and parallel. The opposite angles in any parallelogram are equal.

Sum of angles 1 and 2 = 180°.
Also sum of angles 1 and 3 = 180°.
Hence angle 2 = angle 3.

In the same way angle 1 is shown to equal angle 4.

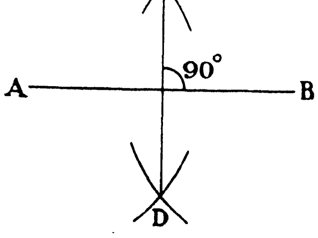


FIG. 53.

To Bisect a Given Straight Line

Let AB be the given line (Fig. 53).

With a radius greater than half the length of AB and with centres at A and B, describe arcs above and below the line.

Join the points where the arcs cross by the line CD. Then CD will bisect AB and the lines CD and AB will be at right angles to each other.

To Bisect a Given Angle

Let angle BAC be the given angle (Fig. 54).

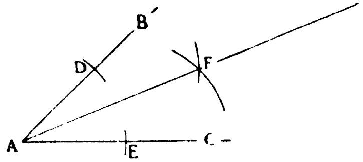


FIG. 54.

With centre A and any radius draw arcs to cut lines AB and AC in points D and E.

With centres D and E describe arcs with the same radius to cut each other at point F.

Join A to F.

The line AF will bisect the angle.

EXERCISE 11

(1) From a point A in a line, set out the angles : (*a*) 43°, (*b*) 117°.

(2) Draw any form of figure bounded by four straight lines. Measure all the internal angles and add them together. What is their sum ?

(3) Draw an angle of 66°. By means of the compasses bisect the angle. Check your result with the protractor.

(4) Construct an angle of 128°. Divide it into four equal parts with the compasses.

(5) On a base 3″ long construct an equilateral triangle.

(6) Construct an isosceles triangle on a base 4″ long, and with the angle at the vertex equal to 80°.

(7) The lengths of the sides of a triangle are 3·4″, 4·8″, and 5·6″. Construct the triangle.

(8) Draw the triangle ABC, the lengths of the sides BA and BC being 6″ and 5″ respectively. The angle at A to be 30°. Show that there are two triangles satisfying these conditions.

(9) Draw a triangle whose sides are 3″, 4″, and 5″ respectively. Measure the angles.

(10) Draw an equilateral triangle of any size. Measure the vertical height of the triangle. Find the ratio of the vertical height to the length of one side. State the answer to the third decimal place.

(11) Construct a triangle two sides of which are 5″ and 6″ and the angle between them is 50°. Measure the length of the remaining side.

(12) Draw an isosceles triangle with a base 6·4″ long and whose vertical height is 3·6″. Measure the length of the sides.

(13) Construct a right-angled triangle in which the hypotenuse is 7½″ long and another side has a length of 6″. What is the length of the other side ?

(14) Draw a line 4¾″ long and divide it into ten equal parts by a geometrical construction.

(15) Construct an equilateral triangle of any size. Produce the base and measure the size of the external angle on either side.

(16) Construct a parallelogram, two of whose adjacent sides are 5″ and 3″ and the included angle is 40°. Measure the other angles.

(17) Two adjacent sides of a parallelogram are 5″ and 4″ and the angle between them is 45°. Construct the parallelogram and measure the lengths of its diagonals.

(18) Draw a parallelogram, two adjacent sides being 4″ and 6″ long and a diagonal 8″ long. Measure the angles.

CHAPTER XII

MENSURATION

Areas and Volumes

A rectangle is a four-sided figure with the opposite sides equal and parallel and the angles right angles.

The area of a rectangle is equal to the length multiplied by the breadth.

Thus in the rectangle shown (Fig. 55) the area is $3 \times 2 = 6$ sq. in—six squares are shown, each of one-inch sides.

Many objects used in engineering are made up of rectangles in the angle form:

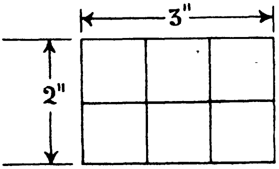


FIG. 55.

Example (1). *Find the area of the* L*-shaped figure* (Fig. 56), *the limbs of the* L *being* $\frac{1}{2}''$ *wide throughout.*

Method. Form two rectangles as shown, Fig. 56.

Area of horizontal rectangle $= 6 \times \frac{1}{2} = 3$ sq. in.
Area of vertical rectangle $= 5\frac{1}{2} \times \frac{1}{2} = 2\frac{3}{4}$ sq. in.
Total area $= 5\frac{3}{4}$ **sq. in.**

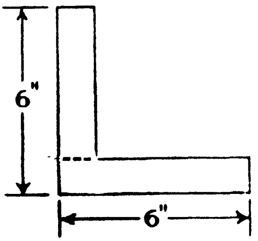


FIG. 56.

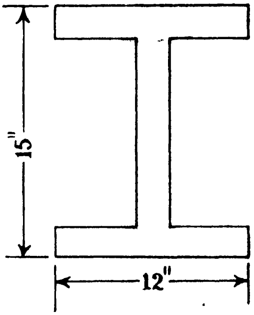


FIG. 57.

Example (2). **The I Section.**

The area of the I *section in Fig.* 57 *is obtained by forming three rectangles. The metal is* $\frac{3}{4}''$ *wide throughout.*

Area of the two flanges $= 2 \times 12 \times \frac{3}{4} = 18$ sq. in.
Area of the web $= 13\frac{1}{2} \times \frac{3}{4}$

$$= \frac{27}{2} \times \frac{3}{4} = \frac{81}{8} = 10\tfrac{1}{8} \text{ sq. in.}$$

Total area $= 18 + 10\frac{1}{8} = 28\frac{1}{8}$ **sq. in.**

The same method may be adopted with the sections Fig. 58 and Fig. 59, or other sections made up of the rectangular forms.

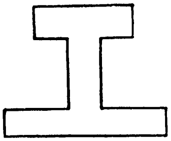


FIG. 58.

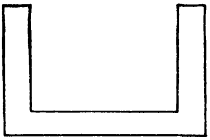


FIG. 59.

Volumes of Solids

In the case of any solid which has the *same cross-section throughout its length,* the volume is given by the expression :

Volume = area of cross-section × length.

Such a solid (Fig. 60) is called a *prism.*

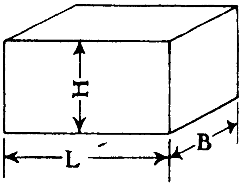


FIG. 60.

In the case of the rectangular solid, the volume is equal to length multiplied by breadth multiplied by height or the

Volume = L × B × H (Fig. 60),
where L = length, B = breadth, H = height,

i.e. the area of one end multiplied by the length.

The volume is given in cubic measure of the unit we use.

For the angle irons we have discussed above, the volume of a given length is obtained by the rule :

Volume = area of section × length.

In the case of the L section above, Fig. 56, the area of the section was found to be 5¾ sq. in. The volume of a 30-ft. length is 5¾ × 30 × 12 cu. in.,

i.e. **2070 cubic inches.**

Example. *An I section has metal ¾" thick throughout. The flanges are 14" wide and the overall height is 16". Find the volume of a 20' length.*

$$\begin{aligned}\text{Area of section} &= 2 \times 14 \times \tfrac{3}{4} + \tfrac{3}{4} \times 14\tfrac{1}{2}\\ &= 31\tfrac{7}{8}\text{ sq. in.}\\ \text{Volume} &= 31\tfrac{7}{8} \times 20 \times 12\\ &= 7650\text{ cu. in.}\end{aligned}$$

If the density of the metal is 0·27 lb. per cubic inch, find its weight.

Weight = 7650 × 0·27 = **2066 lb.**

The Parallelogram

A ***parallelogram*** *has its opposite sides equal and parallel, but its angles may not be right angles. All parallelograms standing on the same base and between the same parallel lines are equal in area* (Fig. 61).

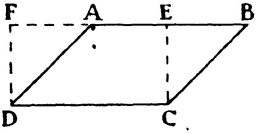


FIG. 61.

In Fig. 61 ABCD is a parallelogram. Draw DF and CE perpendicular to the opposite side, then the triangle AFD is in all respects equal to the triangle BEC; hence the rectangular area EFDC is equal in area to the parallelogram ABCD. Therefore the **area of a parallelogram is equal to the base multiplied by the vertical height.**

In the same way, any other parallelogram standing on the base DC and between the same parallel lines DC and FAEB can be shown to be equal to the rectangle FECD.

Example. *In a parallelogram the base is 4½" long and the vertical height is 3¼". Calculate the area.*

$$\text{Area} = 4\tfrac{1}{2} \times 3\tfrac{1}{4} = \frac{9}{2} \times \frac{13}{4} = \frac{117}{8} = 14\tfrac{5}{8} \text{ sq. in.}$$

The Triangle

All triangles on the same base or on equal length of base and of the same height, i.e. between the same parallel lines, are equal in area.

The triangles ABC, ABD, and ABE are equal in area, for they are on the same base AB and between the same parallel lines, for the lines MM and NN are parallel (Fig. 62). Their heights are equal.

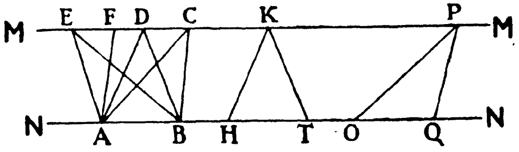


FIG. 62.

We have proved that parallelograms on the same base and between the same parallel lines are equal in area. Now triangle ABC is half of parallelogram ABCF and triangle ABD is half of parallelogram ABDE. But the two parallelograms are equal in area, being on the same base and between the same parallel lines. Hence halves of these equal parallelograms are equal. Therefore triangle ABC is equal to triangle ABD in area.

In a similar manner it may be proved that triangles on equal bases and between the same parallel lines are equal in area.

Triangles ABC, HTK, and OQP (Fig. 62) are equal in area for they are on equal bases and between the same parallel lines.

The student will note that the vertical heights of the triangles are the same.

From this geometry we get the law :

$$\textbf{Area of a triangle} = \frac{\textbf{base length} \times \textbf{vertical height}}{2}.$$

Example (1). *The base of a triangle is 8″ and the vertical height is 5″. Obtain its area.*

$$\text{Area} = \frac{8 \times 5}{2} = 20 \text{ sq. in.}$$

The volume and weight of a bar of triangular section are obtained from :

Volume = area of section × length.
Weight = volume × density.

Example (2). *Obtain the weight of a copper rod 20″ long of the triangular section given in Example* (1).

Volume = 20 × 20 = 400 cu. in.
Density of copper = ·32 lb. per cu. in.
∴ Weight of rod = 400 × ·32 = **128 lb.**

Area of a Triangle when the Lengths of the Three Sides are given.

The area is given by the formula—

$$\textbf{Area} = \sqrt{S(S-a)(S-b)(S-c)},$$

where a, b, and c are the lengths of the three sides and $S = \frac{a+b+c}{2}$, *i.e.* S = half the perimeter of the triangle.

Example. *A triangle has sides whose lengths are* 12″, 13″, and 15″. *Calculate its area.*

Let A be the area.

$$\text{Then } A = \sqrt{S(S-a)(S-b)(S-c)}.$$

Let $a = 12$, $b = 13$, and $c = 15$.

$$\text{Then } S = \frac{12+13+15}{2} = 20.$$

$$\text{Hence } A = \sqrt{20 \times 8 \times 7 \times 5} = \sqrt{5600}.$$

$$\mathbf{A = 74{\cdot}8 \textbf{ sq. in.}}$$

The Trapezium

This is any plane area bounded by four sides where two of them are parallel (Fig. 63).

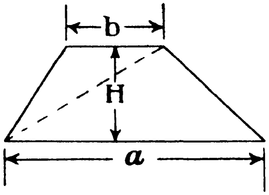


FIG. 63.

The area of a trapezium is given by the formula :

$$\mathbf{A} = \frac{\textbf{sum of parallel sides}}{2} \times \textbf{vertical height.}$$

In the figure shown $A = \frac{(a+b)}{2} \times H.$

By drawing the diagonal as shown, the trapezium is divided into two triangles.

Area of triangle with base $a = \frac{a \times H}{2}$.

Area of triangle with base $b = \frac{b \times H}{2}$.

Total area of trapezium $= \frac{aH}{2} + \frac{bH}{2} = \frac{H}{2}(a+b).$

$$= \frac{(a+b)}{2} \times H.$$

If $a = 14''$, $b = 8''$, and $H = 5''$:

The area $= \frac{(14+8)}{2} \times 5 = 11 \times 5 = 55$ sq. in.

Example. *A bar of metal 20″ long and with cross-section a trapezium, the parallel sides being 5″ and 3″ and the distance between them 2½″. If the density of the metal is 0·3 lb. per cu. in., find the weight of the bar.*

Area of section $= \frac{(5+3)}{2} \times 2\frac{1}{2} = 10$ sq. in.

Volume $= 10 \times 20 = 200$ cu. in.

Weight $= 200 \times 0{\cdot}3 = 60$ lb.

Quadrilateral

In the case of a figure such as is shown in Fig. 64, one should draw a diagonal and thus divide the area into two triangles, then obtain the areas of both triangles and add them together.

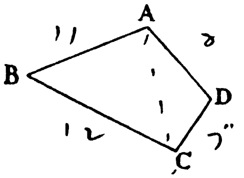


FIG. 64.

In the figure, if the sides have the dimensions : AB = 11", BC = 12", CD = 7", DA = 8", and the diagonal AC = 9", find the area.

Area $= \sqrt{S(S - a)(S - b)(S - c)}$ is the method adopted.

For the triangle ABC, $S = \frac{11 + 12 + 9}{2} = 16.$

$\therefore A = \sqrt{16 \times 5 \times 4 \times 7} = \sqrt{2240} = 47{\cdot}33$ sq. in.

For the triangle ADC, $S = \frac{7 + 8 + 9}{2} = 12.$

Area of this triangle, A $= \sqrt{12 \times 5 \times 4 \times 3} = \sqrt{720}.$
$= 26{\cdot}83$ sq. in.

Total area of the figure $= 47{\cdot}33 + 26{\cdot}83 =$ **74·16 sq. in.**

The Hexagon

The regular hexagon is a six-sided figure, all sides of equal length (Fig. 65).

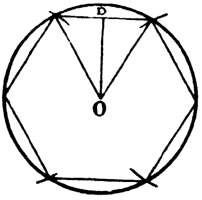


FIG. 65.

As stated previously, it can be constructed by drawing a circle with radius equal to the length of one side of the hexagon and stepping round the circumference with six of these lengths.

It will be noted that if each corner of the hexagon is joined to the centre O, then six equilateral triangles are produced. Hence, six times the area of one of these triangles is the area of the hexagon.

The area of the triangle may be obtained as follows :

Let the length of a side equal 4″, then the area of a triangle

$$= \sqrt{6 \times 2 \times 2 \times 2}$$
$$= \sqrt{48} = 6{\cdot}928 \text{ sq. in.}$$

The height of each triangle as found by later work is 3·464″.

Six times the area of this triangle is the area of the hexagon; *i.e.*

$$= 6 \times 6{\cdot}928 = \mathbf{41{\cdot}568 \text{ sq. in.}}$$

Example (1). *Find the volume and weight of a hexagon-section bar. Length of face 2″, bar length 12″, and density of material* 0·3 *lb. per cubic inch.* (See p. 85 for rule to get volume.)

Area of *hexagon* section $= 6 \times 1{\cdot}732$ sq. in.

= six times the area of one triangle.

Volume in bar $= 1{\cdot}732 \times 12 \times 6 = 124{\cdot}7$ cu. in.

Weight of bar $= 124{\cdot}7 \times {\cdot}3 =$ **37·41 lb.**

Density

The density of a material is the weight of unit volume. It may be stated as the weight of a cubic inch, or the weight of a cubic foot, when these are taken as units. On the metric scale, the density is usually stated as the weight in grams of a cubic centimetre of the material.

To find the density of a metal, an accurate weighing machine is required, such as a physics balance (Fig. 66).

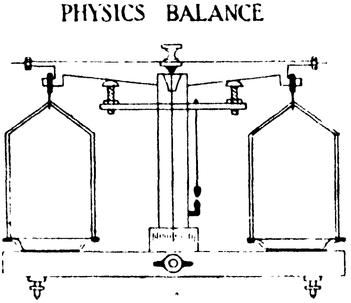


FIG. 66.

Example. Experiment to find the density of a special quality of brass.

A rectangular piece of the brass is taken: length 2·59″, width 0·98″, height 0·91″.

$$\text{Volume} = 2{\cdot}59 \times 0{\cdot}98 \times 0{\cdot}91 \text{ cu. in.}$$
$$= \mathbf{2{\cdot}31} \text{ cu. in.}$$

The weight of the piece by the physics balance is 11 oz. or 0·6875 lb.

$$\text{Density} = \frac{\text{Weight}}{\text{Volume}} = \frac{0{\cdot}6875}{2{\cdot}31} = 0{\cdot}298.$$

i.e. Density = 0·298 *lb. per cubic inch.*

TABLE OF DENSITIES

Name of Material.	Density.	
	lb. per cu. ft.	lb. per cu. in.
Water	62·3	0·036
Aluminium	161·7	0·094
Antimony	417	0·24
Bismuth	613	0·35
Brass (average)	503	0·3
Copper	545	0·32
Cast iron	465	0·265
Wrought iron	483	0·28
Mild steel	480	0·27
Lead	708	0·41
Nickel	547	0·31
Petroleum oil	55	0·032
Tin	452	0·26
Zinc	445	0·258

CONVERSION TABLE, OUNCES TO THE DECIMAL FRACTION OF ONE POUND

Ounces.	*Decimal Fraction of* 1 *lb.*
½	·03125
1	·0625
2	·125
3	·1875
4	·25
5	·3125
6	·375
7	·4375
8	·5
9	·5625
10	·625
11	·6875
12	·75
13	·8125
14	·875
15	·9375
16	1·0000

THE CYLINDER

The cylinder is a solid of the same circular cross-section throughout its length (Fig. 67).

The circumference of a cylinder is given by the formula : $C = \pi D$, where C is the circumference and D is the diameter.

The area of the curved surface = circumference × length.

In sheet-metal work the cylinder is made from a rectangular sheet of metal.

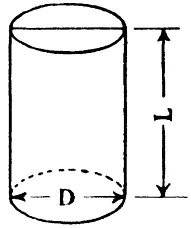


FIG. 67.

Example. *The diameter of a cylinder is 5″ and the vertical height is 8″. Find the area of its curved surface.*

$$\textbf{Area} = \frac{22}{7} \times 5 \times 8 = \frac{880}{7} = 125{\cdot}9 \text{ sq. in.}$$

Volume

Since this solid has the same section throughout, its volume is found by the rule given on p. 85.

Whence **volume of a cylinder** = **area of base** × **length.**

Now the area of a circle $= \pi r^2$ or $\dfrac{\pi D^2}{4}$.

∴ If h = height of cylinder and V is the volume—

$$V = \pi r^2 h \text{ or } \frac{\pi D^2 h}{4}.$$

Using the latter formula, the volume in the above cylinder—

$$= \frac{22}{28} \times 25 \times 8 = 157\tfrac{1}{7} \text{ cu. in.}$$

Weight of a cylinder is the volume × density.

If the above cylinder were made of mild steel of density 0·28 lb. per cu. in., the weight equals $157\frac{1}{7} \times {\cdot}28 = 44$ lb.

Example. *A steel shaft is 3½″ diameter and 20′ long. Calculate its weight. Density = ·28 lb. per cu. in.*

$$\text{Volume} = \frac{22}{28} \times (3\tfrac{1}{2})^2 \times 20 \times 12 \text{ cu. in.}$$

$$= 2310 \text{ cu. in.}$$

$$\text{Weight} = 2310 \times {\cdot}28 = \mathbf{646{\cdot}8 \text{ lb.}}$$

The Hollow Circle or the Annulus

When two circles are drawn with the same centre, the area between them is called an **annulus** (Fig. 68).

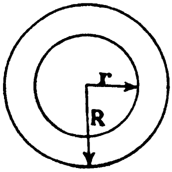


FIG. 68.

If R and r are the radii of the outer and inner circles, then:

$$\text{Area of the annulus} = \pi R^2 - \pi r^2$$
$$= \pi(R^2 - r^2).$$

If the diameters are given, the area :

$$A = \frac{\pi}{4}(D^2 - d^2)$$

where D is the outer diameter and d the smaller diameter.

Let the outer diameter be 12″ and the inner one 10″, then—

$$A = \frac{22}{28}(12^2 - 10^2)$$

$$\textit{i.e. } A = \frac{22}{28} \times 44 = 34\tfrac{4}{7} \text{ sq. in.}$$

Volume of a Tube

Let the above section (Fig. 68) be that of a tube 9′ long. The volume is equal to the cross-section area multiplied by the length. Thus:

$$\text{Volume} = 34\tfrac{4}{7} \times 108 \text{ cu. in.}$$
$$= 3733\tfrac{5}{7} \text{ cu. in.}$$

Example (1). *Find the weight of* 5000 *brass washers,* 2″ *outer diameter and* 1″ *inside diameter, and* ⅛″ *thick. Density of brass equals* 0·3 *lb. per cu. in.*

$$\text{Volume} = 5000 \times \frac{22}{28}(2^2 - 1^2) \times \tfrac{1}{8} \text{ cu. in.}$$

$$\text{Weight} = 5000 \times \frac{22}{28} \times \frac{3}{8} \times \cdot 3 = \mathbf{442\ lb.}$$

Example (2). *A hollow shaft* 14″ *external diameter and* 7″ *internal diameter and* 20′ *long is made of steel having a density of* 0·28 *lb. per cu. in. What is its weight?*

$$\text{Area of section} = \frac{22}{28}(14^2 - 7^2).$$

$$\text{Volume of the shaft} = \frac{22}{28}(147) \times 12 \times 20.$$

$$\text{Weight of the shaft} = \frac{22}{28} \times 147 \times 12 \times 20 \times 0{\cdot}28 \text{ lb.}$$
$$= 7762 \text{ lb.}$$
$$= \frac{7762}{2240} = \mathbf{3{\cdot}465\ tons.}$$

Sector of a Circle

A sector is a portion of a circle enclosed between two radii and the arc they cut off.

In a sector, the length of the arc is proportional to the angle between the radii (Fig. 69).

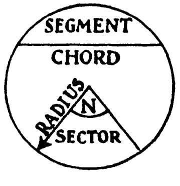


Fig. 69.

There are 360° in a complete revolution of the radius.

Hence we could have 360 sectors of equal size in a circle.

Therefore the **length of the arc for each one** of these small sectors

$$= \frac{\text{circumference}}{360}.$$

Similarly, the **area of one of these small sectors** $= \dfrac{\text{area of the circle}}{360}.$

If there is an angle of N degrees in the sector, then—

$$\text{length of arc} = \frac{\pi d \text{N}}{360}.$$

Similarly :

$$\text{area of the sector} = \frac{\pi d^2 \text{N}}{4 \times 360}.$$

Example (1). *Let the diameter* $d = 9''$ *and* N = 70°. *Find the length of arc and also the area of the sector.*

$$\text{Length of arc} = \frac{22}{7} \times \frac{9}{360} \times 70 = 5\tfrac{1}{2}.$$

$$\text{Area of the sector} = \frac{22}{28} \times 9 \times 9 \times \frac{70}{360}.$$

$$= 12\tfrac{3}{8} \text{ sq. in.}$$

Example (2). *Find the area of the sector of a circle,* $3\frac{1}{2}''$ *radius, and the angle between the radii is* 220°. *Also find the length of the arc.*

$$\text{Area of sector} = \frac{220}{360} \times \frac{22}{7} \times 3\tfrac{1}{2} \times 3\tfrac{1}{2}.$$

$$= 23{\cdot}5 \text{ sq. in.}$$

$$\text{Length of arc} = \frac{220}{360} \times \frac{22}{7} \times 7 = 13{\cdot}5''.$$

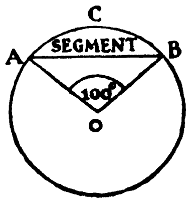


FIG. 70.

Area of a Segment of a Circle

A segment of a circle is a portion cut off by a chord.

Let the radius OA or OB = $3\frac{1}{2}''$ and find the area of the segment (Fig. 70).

Method. First obtain the area of the sector OACB, and then take away the area of the triangle OAB.

THE SPHERE

The **total surface area of a sphere** is given by the formula :

$$\mathbf{A} = 4\pi r^2 \text{ or } \pi d^2$$

where r is the radius, d the diameter, and A the area.

We note that the area of the sphere is just four times the area of a disc of the same diameter.

The **volume of a sphere** is given by the formula :

$$\mathbf{V} = \frac{4}{3}\pi r^3 \text{ or by } \frac{\pi d^3}{6}.$$

Let the diameter be 6″, then—

$$\text{the area} = \frac{22}{7} \times 36 = 113\tfrac{1}{7} \text{ sq. in.}$$

$$\text{the volume} = \frac{22}{7} \times \frac{6^3}{6} = 113\tfrac{1}{7} \text{ cu. in.}$$

By a coincidence the area and volume are given by the same number; of course, this is not so generally.

Example. *Find the surface area and volume of a sphere 3½″ diameter.*

$$\text{Surface area} = \pi d^2 = \frac{\overset{11}{\cancel{22}}}{\underset{\cancel{2}}{\cancel{7}}} \times \cancel{3\tfrac{1}{2}} \times 3\tfrac{1}{2}$$

$$= 11 \times 3\tfrac{1}{2} = 38\tfrac{1}{2} \text{ sq. in.}$$

$$\text{Volume} = \frac{\pi}{6}d^3 = \frac{22}{28} \times \frac{3\tfrac{1}{2} \times 3\tfrac{1}{2} \times 3\tfrac{1}{2}}{6}$$

$$= 22\tfrac{11}{24} \text{ cu. in.}$$

Portion of an Annulus

The area of a portion of an annulus (Fig. 71) is equal to the **area of the larger sector minus the area of the smaller.**

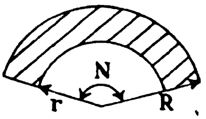


FIG. 71.

Let N be the number of degrees in the sector, then—

$$\text{Area of the larger sector} = \frac{\pi R^2 N}{360}.$$

$$\text{,, \quad ,, \quad smaller \quad ,,} = \frac{\pi r^2 N}{360}.$$

∴ Area of the portion of sector, shown sectioned.

$$= \frac{\pi R^2 N}{360} - \frac{\pi r^2 N}{360}$$

$$= \frac{\pi N}{360}(R^2 - r^2).$$

Let R = 6″, r = 4″, and N = 150°, then the area:

$$= \frac{22}{7} \times \frac{150}{360}(36 - 16) \text{ sq. in.}$$

$$= \frac{22}{7} \times \frac{5}{12} \times 20 = 26\tfrac{4}{21} \text{ sq. in.}$$

EXERCISE 12

(1) A rectangular block of metal is 1·46″ long, 3·5″ wide, and 0·49″ thick. Find its weight if the density is 0·28 lb. per cubic inch.

(2) An angle iron of the L shape, 15′ long, has limbs 6″ long. The metal is ½″ thick throughout. If the density is ·28 lb. per cubic inch, find its weight.

(3) What is the area of a parallelogram of base length 12″ and vertical height 6·5″ ?

(4) A triangle has a base 14·4″ long and a vertical height of 12″. What is its area ?

(5) Find the area of a triangle whose sides are 4·75″, 2·15″, and 5·32″ in length.

(6) Obtain the area of a triangle whose two sides are 41·2″ and 27·3″, and the angle between them is 47°. (Construct the triangle and measure the vertical height.)

(7) Obtain the area of a parallelogram if two adjacent sides measure 12·25″ and 10·2″, with the included angle 40°. (Construct the parallelogram and find the vertical height.)

(8) Find the area of a trapezium, the parallel sides being 8·6″ and 3·2″ long, and the vertical height 4·1″.

(9) Find the area of a parallelogram having two adjacent sides 10″ and 8″, with an included angle of 53°.

(10) Construct an isosceles triangle having an area of 7½ sq. in. and a base length of 6″.

(11) What is the length of the side of a square equal in area to a parallelogram, 3″ base and 1″ vertical height ?

(12) Find the length of the side of a square drawn in a circle 8″ diameter, with the corners of the square on the circumference.

(13) Find the volume and surface area of a cylinder whose diameter is 15″ and length 22½″.

(14) Find the volume of a cylinder 7·6 cm. long and 3·16 cm. diameter.

(15) A cylinder is 2·52 cm. diameter and 10·24 cm. long. Calculate its volume.

(16) Twelve holes ½″ diameter are drilled through a circular steel plate 12″ diameter and 1″ thick. If the density of the steel is 480 lb. per cubic foot, find the weight of the plate.

(17) Find the number of cubic inches in a litre. (One inch = 2·54 cm.)

(18) A rectangular piece of metal—length 51·8 cm., breadth 32·5 cm., height 27·7 cm. (*a*) Express the volume in cubic feet. (*b*) State its weight if the density is 500 lb. per cubic foot.

(19) A hollow cylinder has an external diameter of 14″, an internal diameter 12″, and a length of 9″. Find its volume.

(20) The length of a hollow cylinder of lead is 95 cm., its external and internal radii are 21 and 16 cm. respectively. The density is 11·28 grm. per cubic cm. Find its weight in Kilograms.

(21) Find the length of the side of a square drawn in a circle 2′ diameter with the corners of the square on the circumference.

(22) The radius of a sphere is 3·5″. Calculate its volume.

(23) Find the volume of a sphere whose diameter is 1·56 cm.

(24) The volume of a sphere has to be 5·2 cu. in. Find its diameter.

(25) The crank of an engine makes 2000 revolutions a minute. Find its angular velocity in radians per second. (There are 2π radians in a revolution.)

(26) Find the length of the arc in a sector of a circle 5″ diameter if the angle between the radii is 30°.

(27) A driving belt passes over a pulley and the angle of lap is 165°. Find the length of the belt in contact with the pulley 1′ 3″ diameter.

(28) A belt passes over the engine flywheel rim 7′ diameter, which revolves at an angular velocity of 3·2 radians per second. Find the belt speed in feet per minute.

(29) The diameter of a steel plate is 2′, and a sector containing an angle of 140° is cut from it. The plate is made from sheet metal which is 10 lb. per square foot in weight. What is the weight of the sector ?

(30) A cast-iron sphere is 8·7 lb. in weight. Find its diameter. Density of cast iron is 0·26 lb. per cubic inch.

(31) Find the area and circumference of a circle of 4″ diameter.

(32) What is the diameter of a circle whose area is 20 sq. in. ?

(33) A boiler has 150 tubes, 16′ long and 3″ diameter. What is the total cross-section area of the tubes ? Find the total area of the tubes for heating purposes ; *i.e.* the outside area of the tubes.

(34) What is the weight of 100′ of copper wire of diameter ·128″, the density of copper being 0·32 lb. per cubic inch ?

(35) A cast-iron pipe is 5½″ external diameter and 4½″ internal diameter. Calculate the cross-section area of the metal.

(36) A sphere is 8″ diameter. What are its area and its volume ?

(37) A casting is a hollow rectangle in section and 12′ long. The cross-section is 18″ wide and 20″ deep. If the weight of 1 cu. ft. of the metal is 490 lb., calculate the weight of the casting when the thickness of the metal throughout the section is 1½″.

CHAPTER XIII

SIMILAR TRIANGLES AND TRIGONOMETRY

JUST as the plan or elevation on a blue-print of a machine piece is similar in outline to the actual machine piece, so triangles of different sizes—but equiangular—are similar. Equiangular means that the corresponding angles of the triangles are equal, each to each. Also, as in the case of the machine drawing, any pair of similar measurements are in the same ratio as the corresponding pair of measurements on the machine, so in similar triangles the ratio of any pair of sides in one triangle is equal to the ratio of the similar sides in the other triangle. Calculations in **trigonometry** are based on this principle.

The word Ratio, as was shown in Chapter V, means the fraction formed by two numbers, *e.g.* the ratio of 5 to 8 is the fraction $\frac{5}{8}$.

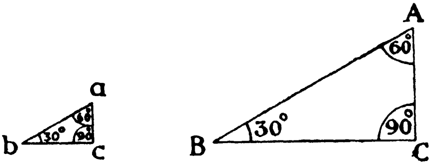


FIG. 72.

Let the triangles ABC, *abc* (Fig. 72), be similar triangles.

Consider the lengths of the sides of these triangles. By definition—

$$\frac{ca}{bc} = \frac{CA}{BC}$$

$$\frac{ca}{ba} = \frac{CA}{BA}$$

$$\frac{bc}{ba} = \frac{BC}{BA}$$

Note.—In a right-angled triangle the longest side, *i.e.* the side which faces the right angle, is always called the Hypotenuse.

In right-angled triangles, names are given to these ratios as follows:

$\frac{CA}{BC}$ is called the **tangent** of the angle B,

$\frac{CA}{BA}$ is called the **sine** of the angle B,

And $\frac{BC}{BA}$ is called the **cosine** of the angle B.

That is, referring to angle B,

$$\frac{\text{Side opposite the angle}}{\text{Side adjacent to the angle}} = \textbf{tangent} \text{ of the angle.}$$

$$\frac{\text{Side opposite the angle}}{\text{Hypotenuse}} = \textbf{sine} \text{ of the angle.}$$

$$\frac{\text{Side adjacent to the angle}}{\text{Hypotenuse}} = \textbf{cosine} \text{ of the angle.}$$

These names are abbreviated to tan, sin, and cos.

Thus in the smaller triangle:

$$\tan a = \frac{cb}{ac}$$

$$\sin a = \frac{cb}{ab}$$

$$\cos a = \frac{ac}{ab}$$

The values of the sines, cosines, and tangents of all angles from 0° to 90° are given in the tables at the end of the book.

Taking the angles given in the triangles above (Fig. 72);

$$\sin 30° = \cdot 5, \quad \cos 30° = \cdot 866, \tan 30° = \cdot 5774.$$
$$\sin 60° = \cdot 866, \cos 60° = \cdot 5, \quad \tan 60° = 1\cdot 7321.$$

Hence, whatever size these triangles (Fig. 72) might be, the values of the ratios sin, cos, and tan will be those stated providing the angles are 30° and 60°.

Similarly for triangles containing other angles—the values of the ratios may be found in the tables at the end of the book.

By means of this mathematical principle calculations with respect to angles and distances can be made.

The Tangent

One of the earliest examples that we know in history of the practical applications of geometry was the problem of finding the height of one of the Egyptian pyramids. This was solved by Thales, the Greek mathematician. For this purpose he used the property of similar triangles which is stated on p. 98, and he did it in this way.

He observed the length of the shadow of the pyramid and, at the same time, that of a stick, AB, placed vertically into the ground at the end of the shadow of the pyramid (Fig. 73). QB represents the length of the shadow of the

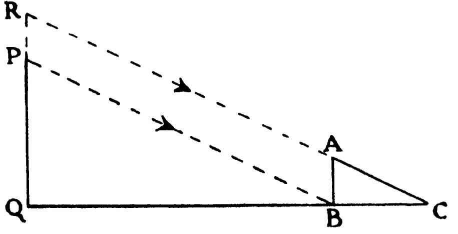


FIG. 73.

pyramid, and BC that of the stick. Then he said, "*The height of the pyramid is to the length of the stick, as the length of the shadow of the pyramid is to the length of the shadow of the stick.*"

i.e. in Fig. 73
$$\frac{PQ}{AB} = \frac{QB}{BC}.$$

Then QB, AB, and BC being known we can find PQ.

The solution is independent of the length of the stick AB because if this be changed the length of its shadow will be changed proportionally.

We therefore can make this important general deduction :

For the given angle ACB *the ratio* $\frac{AB}{BC}$ *remains constant whatever the length of* AB.

This ratio can therefore be calculated beforehand whatever the size of the angle ACB. If this be done there is no necessity to use the stick, because knowing the angle and the value of the ratio, when we have measured the length of QB, we can easily calculate PQ. Thus if the altitude were found be to 64° and the value of the ratio for this angle had been previously calculated to be 2·05, then we have—

$$\frac{PQ}{QB} = 2{\cdot}05$$

and
$$PQ = QB \times 2{\cdot}05.$$

This ratio is that which we have called a *tangent*, see page 99. Hence for every angle there is a definite number giving the value of the tangent. Similarly every angle has its own sine and cosine.

Example (1). *In the triangle* ABC (Fig. 74), AC = 10 *units and* CB = 7 *units. Calculate the angles* A *and* B.

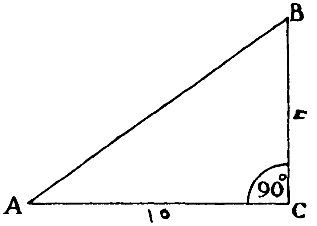


FIG. 74.

$$\text{Tan A} = \frac{CB}{AC} = \tfrac{7}{10} = {\cdot}7.$$
$$\text{Tan A} = {\cdot}7.$$

From the tables we find that to the nearest degree, angle A = 35°.

∴ Angle B = 90° − 35° = 55°.

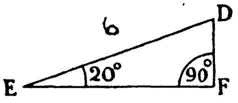


FIG. 75.

Example (2). *In the triangle* DEF (Fig. 75), DE = 6 *units, angle* E = 20°. *Find the remaining angle and the lengths* EF *and* FD.

Angle D = 90° − 20° = 70°.

$$\frac{FD}{DE} = \sin 20^\circ, \therefore \frac{FD}{6} = \cdot 342, \quad \therefore FD = 2\cdot 052 \text{ units.}$$

$$\frac{EF}{DE} = \cos 20^\circ, \therefore \frac{EF}{6} = \cdot 9397, \therefore EF = \mathbf{5\cdot 6382 \text{ units.}}$$

Example (3). *In a rectangle find the angles between the diagonals and the adjacent sides—the latter being 8″ and 5″. Also obtain the length of the diagonals and the angles between them.*

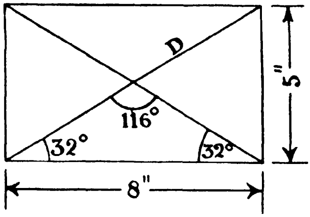


FIG. 76.

In Fig. 76, let θ be the angle between the 8″ side and the diagonals.

$$\tan \theta = \tfrac{5}{8}$$
$$= \cdot 625.$$
$$\therefore \hat{\theta} = 32^\circ.$$

To find the length of D :

$$\frac{5}{D} = \sin 32^\circ = \cdot 5299.$$

$$\therefore D = \frac{5}{\cdot 5299} = 9\cdot 434''.$$

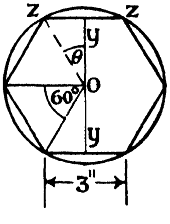


FIG. 77.

Both diagonals make an angle of 32° with the 8″ side ; hence the angle between the diagonals is 180° − 64° = **116°.**

Example (4). *In a hexagon, length of side* 3″, *what is the distance across the flats—that is, the distance* yy (Fig. 77) ?

Now *yy* is at right angles to *zz*. ∴ triangle *oyz* is a right-angled triangle.

$$\therefore \frac{oy}{oz} = \cos \theta.$$

Being a hexagon the radius $oz = 3''$, also the angle subtended at the centre by a side of the hexagon $= 60°$.

Hence $\theta = 30°$.

$$\therefore \frac{oy}{3} = \cos 30°. \quad \therefore \frac{oy}{3} = \cdot 866.$$

$$oy = 3 \times \cdot 866 = 2\cdot 598''.$$

$$\text{Hence } yy = 2 \times 2\cdot 598 = 5\cdot 196''.$$

Example (5). *Fig.* 78 *shows a* **V** *notch. The angle of the* **V** *must be* 40°. *What must be the dimension x ?*

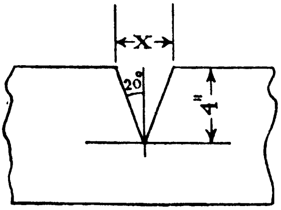


FIG. 78.

From the figure—

$$\tan 20° = \frac{\frac{1}{2}x}{4} = \frac{x}{8}$$

$$\tan 20° = \cdot 364.$$

$$\therefore \quad \frac{x}{8} = \cdot 364.$$

$$\text{Hence } x = 2\cdot 912''.$$

Example (6). *Fig.* 79 *shows the tapered end of a machine spindle. Calculate the angle of taper.*

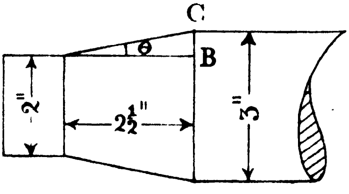


FIG. 79.

Method.—First find the angle θ and then double the answer. In the triangle, BC $= \frac{1}{2}''$.

$$\therefore \tan\theta = \frac{\frac{1}{2}}{2\frac{1}{2}} = \frac{1}{5} = \cdot 2$$

$$\therefore \theta = 11° \, 19'.$$

Hence the total angle of taper $= (11° \, 19') \times 2$

$= \mathbf{22° \, 38'}$.

Example (7). *In Fig. 80, find the width of the* **V** *at the top.*

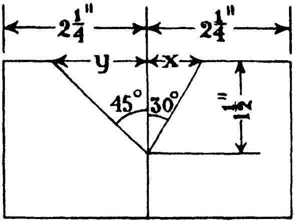


FIG. 80.

$$\text{From the figure } \frac{x}{1\frac{1}{2}} = \tan 30^\circ = \cdot 5774.$$

$$\therefore x = \cdot 5774 \times 1\tfrac{1}{2} = \cdot 8661.$$

$$\text{Also } \frac{y}{1\frac{1}{2}} = \tan 45^\circ = 1. \quad \therefore y = 1\tfrac{1}{2}.$$

$$\therefore x + y = \cdot 8661 + 1\tfrac{1}{2} = 2\cdot 3661''.$$

EXERCISE 13

(1) Write down the values of the following ratios :

- (*a*) Sines : 30°, 60°, 0°, 90°, 45°, 22° 30′, 67° 30′.
- (*b*) Cosines of the above angles.
- (*c*) Tangents of the above angles.

(2) Find the sin, cos, and tan of 14° 11′.

(3) Draw a right-angled triangle with one of the acute angles 35°.

(4) In the triangle of question (3), measure the lengths of the sides and calculate the values of the ratios : tan 35°, sin 35°, cos 55°, sin 55°. Check your results with the tables at the end of the book.

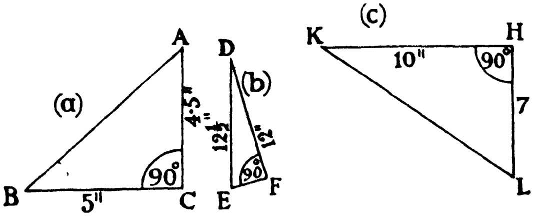


FIG. 81.

(5) In the three right-angled triangles (Fig. 81) find the sizes of the acute angles.

(6) In the right-angled triangles (Fig. 82) obtain the values of the acute angles.

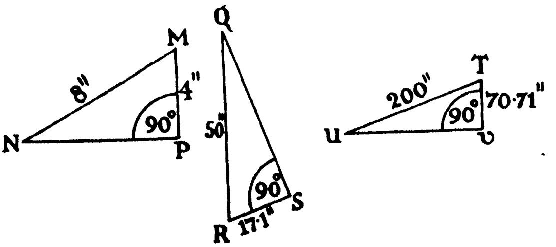


FIG. 82.

(7) Two points A and B are in a straight line and the distance between them is 15″. Another point C is located, 12″ from A and 9″ from B. Draw the triangle. Measure angle C. Calculate the acute angles at A and B.

(8) Two adjacent sides of a parallelogram are 10″ and 12″ and the angle between them is 60″. If the 12″ side acts as base, what is the vertical height of the parallelogram ?

(9) Four holes equally spaced have to be drilled on the circumference of a circle 4″ diameter. What is the chordal distance between two centres ? Also what is the angle between the chord and a diameter from the end of the chord ?

(10) A sector of a circle 4″ radius contains an angle of 80° between the radii. Calculate the length of the arc and also its area.

(11) In an equilateral triangle whose sides are 1·7″ long, what is the diameter of the inscribed circle ?

(12) A cone has a base diameter of 3″ and the vertical height is 4″. What is the angle at the vertex ?

(13) A cone has a base diameter of 6½″ and an angle at the vertex 55° 2′. Find the perpendicular height of the cone.

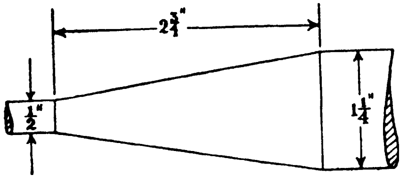


FIG. 83.

(14) In the link with a taper portion shown in Fig. 83 :
- (*a*) Calculate the amount of taper per inch length and per foot length, and
- (*b*) Calculate the angle of taper to the nearest degree.

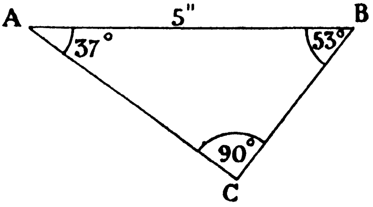


FIG. 84.

(15) The set-out on a piece of flat work is shown in Fig. 84. Calculate the lengths of lines AC and BC, when AB is 5″ long.

CHAPTER XIV

FORCE

The earliest idea of a force which one appreciates is that of lifting an object vertically from the floor. To hold the object freely off the ground one must exert a force. If the object is a 56-lb. weight, then an upward force of 56 lb. must be exerted. Fig. 85 shows the application of this force.

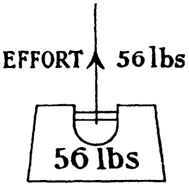


Fig. 85.

In Fig. 85 a straight line is drawn to represent the force or the effort which is acting vertically **upwards.** This vertical force must necessarily balance the weight which acts vertically **downwards.** If it is at rest, however an object is supported, whether on the horizontal plane or on an incline, the weight of it acts vertically downwards. *This vertical downward force acting on every object, and which is equal to the weight of the object, is called* **the force of gravity.** This *force of gravity,* acting on all objects of whatever material, is due to the **attraction of the earth** on the object. It is this attraction which causes an object to have weight. Thus the attractive force of the earth for an object is equal to the weight of it, and the direction of the attraction is always vertically downwards.

Sir Isaac Newton expounded **three laws of motion** as the foundation of knowledge which involves force.

1st law. *Every object perseveres in its state of rest, or of uniform motion in a straight line, unless impelled by impressed force to change.*

2nd law. *The rate of change of motion is proportional to the impressed force.*

3rd law. *To every acting force there is a reacting one equal in amount but opposite in direction.*

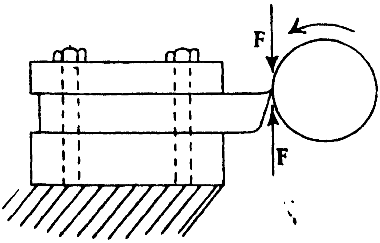


Fig. 86.

From these laws we may gather a definition of what is meant by **Force.** *Force is that which tends to produce motion in an object if it is at rest, or to change the motion of an object if it is moving.*

Other Forces

Besides the force of gravity, the engineer must contend with other forces which may be applied in some way to various parts of machines or structures, *e.g.* **cutting force** at a lathe tool point.

Fig. 86 illustrates the tool held in the tool box with the work turning on to the point. The tool gives an **upward cutting force F lb.**, which overcomes the strength of the material and takes metal off the piece. To remove metal by machining, the tool must penetrate the surface of the metal. The **resistance to surface penetration** of the metal depends on the relative indentation hardness of the tool and metal. Obviously, the cutting end of the tool must be of much greater hardness than the metal to be cut or machined.

Transmission of Force

A simple example illustrating the transmission of force may be taken from a youthful sport.

Take the case of the sport known as *Tug of War*, where a rope is being pulled by a team of youths at each end.

Let AB (Fig. 87) be the rope which is just in balance with the teams at both A and B pulling with a force of 300 lb. On teaching a class of first-year students I ask, "What would be the pull on the spring-balance S, if one were attached in the middle and pulled at by the two parts of the rope?" A discussion usually takes place amongst the students, some saying 300 lb. but many saying 600 lb.

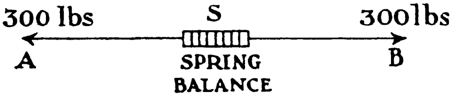


Fig. 87.

The student should note that to balance a force of 300 lb. at A requires a force of 300 lb. acting in the opposite direction. Moreover, this force B, reacting to the force A, may be placed at any section of the rope from A to B and thus the force shown on the spring-balance would be 300 lb. This is in accordance with **Newton's 3rd law**, *i.e. To every acting force there must be a reacting force equal in amount to the acting one but opposite in direction.*

The student should note particularly that in the case of the tug-of-war rope shown above, every part of the rope from A to B is supporting the full load of 300 lb. Likewise, when any link of a machine or structure is loaded by a pull at one end then an equal reacting pull is required at the other end in the opposite direction. Also every cross-section in the link is loaded in a similar manner, and this causes stresses between the particles within the link sections.

A simple experiment illustrative of this law is shown in Fig. 88. Several spring-balances are placed end to end on a horizontal table. The end A is clamped and a pull of P lb. is applied at the end B. **All the spring balances will read P lb., illustrating that the force P lb. is transmitted from B to A, and that at any section between A and B the force is P lb.**

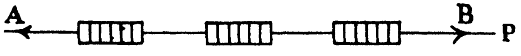


Fig. 88.

Terms used in Engineering

Tension, Compression, Shearing Load

A machine link, or the link in a structure which is subject to a pull at each end, is said to be in **tension** or under a **tensile load.** If the link is under a crushing load, it is said to be in **compression.** If at any section of the link one part is being pushed off the other, it is said to be under a **shearing load** (Fig. 89).

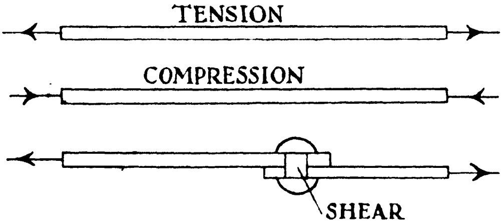


Fig. 89.

It has been stated before that a link loaded in tension by a force P lb. or tons is loaded at every cross-section from end to end of the link by the same force. This causes a state of **stress** to be set up between the particles of the material of which the link is made, that is the particles tend to separate from one another. Thus, in the case of tension we get **tensile stress** ; in the case of compression, **compressive stress** ; while in the case of shear, **shear stress.**

Transmission by Belt

The force in the belt shown in Fig. 90 illustrates the transmission of force from one shaft to another.

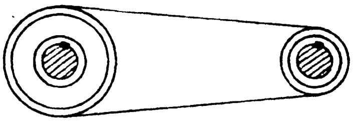


Fig. 90.

However a force is applied by part of a machine, it is transmitted throughout that machine. In Fig. 86, a force is transmitted through the tool box to the bed of the machine. In the other direction the cutting force at the tool point is transmitted through the work to the lathe chuck and thence through the spindle to the belt driving it.

In Fig. 90 the pull in the belt is transmitted through the pulley to the key which fastens the pulley to the shaft. In the key there is a shearing force, which is used to turn the shaft round, introducing a **torsional shearing force** in the shaft. Also in Fig. 86 with the tool we note that there is a tendency to bend the tool.

However forces are applied to objects a state of stress is set up between the particles of the material—for example, the particles are pulling at one another and tend to separate. When the force is big enough the particles do separate—for example, in turning, milling, or in the case of a planing job, the particles are sheared from their neighbours.

Loading of Machines and Structures

Machines and structures may be loaded in a variety of ways. There is the direct way of loading by weights, but the loading of machine tools and hand tools is not so obvious. Generally speaking, the load applied to a machine is transmitted from the point of application throughout the machine. Take the case of a **lathe—the load is applied at the tool point, the thrust of the tool is transmitted to the centres ; the cutting force to the chuck or carrier,** thence to the driving pulley and belt, and so on.

Stresses, and Intensity of Stress

As a simple example AB (Fig. 91) represents a steel rod supporting a pulling force at each end of F lb. This load of F lb. is transmitted along the rod from end to end ; if a cross-section is taken at any point from A to B, that section is supporting F lb. The particles of steel across the section are pulling at one another and this is what is meant by **stress within the material.** The pull on a square inch of particles is called the **intensity of stress.** In order to calculate the intensity of stress, divide the load F lb. by the cross-section area in square inches.

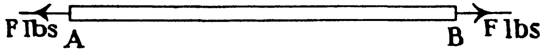


FIG. 91.

Thus, **intensity of stress** $= \dfrac{\textbf{load}}{\textbf{section area}}$.

Example (1). *A straight link rectangular in section has a pull of* 15 *tons applied at the ends. The link is* 3″ *wide and* ⅝″ *thick. Calculate the intensity of stress.*

$$\text{Intensity of stress} = \frac{\text{load}}{\text{section area}} = \frac{15}{3 \times \frac{5}{8}}.$$
$$= 15 \times \tfrac{8}{15} = \mathbf{8\ tons\ per\ sq.\ in.}$$

Example (2). *If the above link had a rivet hole* 1″ *diameter bored through it, calculate the intensity of stress at the section across the centre of the rivet hole.*

Fig. 92 illustrates the section, with metal removed.
Section area remaining = **2 × ⅝ sq. in.**

$$\therefore \textbf{Intensity of stress} = \frac{15}{2 \times \frac{5}{8}} = \frac{15 \times 8}{10}$$
$$= \mathbf{12\ tons\ per\ sq.\ in.}$$

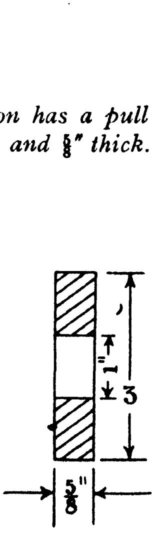


FIG. 92.

Example (3). *A rivet,* ¾″ *diameter, sustains a shearing load of* 2 *tons. Calculate the intensity of shear stress. The rivet is in single shear.*

$$\text{Intensity of shear stress} = \frac{2}{\frac{22}{4 \times 7} \times \left(\frac{3}{4}\right)^2} = \cancel{2} \times \frac{28}{\underset{11}{\cancel{22}}} \times \frac{16}{9}$$

$$= 4{\cdot}53 \text{ tons per sq. in.}$$

Example (4). *A cast-iron pillar,* 16 *sq. in. in section area, supports a crushing load of* 80 *tons. Calculate the intensity of compressive stress.*

$$\text{Intensity of compressive stress} = \frac{80}{16} = 5 \text{ tons per sq. in.}$$

Example (5). *A rivet,* 1¼″ *diameter, has a shearing load in it of* 5½ *tons. Calculate the intensity of shear stress.*

$$\text{Intensity of stress} = \frac{5\frac{1}{2}}{\frac{22}{28} \times \left(\frac{5}{4}\right)^2} = \frac{5\frac{1}{2} \times 28 \times 16}{22 \times 25}$$

$$= 4{\cdot}48 \text{ tons per sq. in.}$$

Kinds of Stress

There are three direct kinds of loading a machine piece or structure :

(1) A pulling load. This introduces a stress called **Tensile Stress.**

(2) A crushing load. This produces a **Compressive Stress** within the material.

(3) A shearing load, and this gives a **Shear Stress** within the material. Also a **twist** on a shaft, spindle, etc., gives a shear stress in the material.

But there may be combinations of these stresses in any one link of a machine or structure.

Tie and Strut

When a link is subject to a tensile load, it is called a **Tie,** and when subject to a compressive load it is called a **Strut.**

Example. *A tie, rectangular in section,* 3″ × ½″, *is subject to a pull of* 4½ *tons. Calculate the intensity of tensile stress.*

$$\text{Intensity of stress} = \frac{\text{load}}{\text{section area}} = \frac{4\frac{1}{2}}{3 \times \frac{1}{2}}$$

$$= \frac{4\frac{1}{2}}{1\frac{1}{2}} = 3 \text{ tons per sq. in.}$$

Strength of Materials

All constructive metals are aggregates of small crystals or particles, and it is the **cohesion** between these crystals which causes the strength of the material. We have seen that when an object is loaded, these particles tend to separate ; this we have called stress. Thus, whether an object breaks or not is a question whether the stress is greater than the cohesion between the particles or crystals. In the operations of turning, milling, planing, or drilling, the stress is obviously greater than cohesion or strength.

As in the case of **intensity of stress**, engineers denote the—

> **Strength** *of a material as the amount of cohesion on a square inch of any cross-section.*

For instance, the strength of mild steel is said to be 28 tons per sq. in. ; this means that the amount of cohesion on a square inch of particles is 28 tons.

TABLE OF STRENGTHS. TONS PER SQUARE INCH

Name of Material.	*Tensile.*	*Compressive.*	*Shear.*
Bessemer steel	52·2	—	35·2
Cast steel	28·04		27·6
Copper	14·0	—	11·6
Muntz metal	25·46	—	18·6
Gun metal	13·68	—	12·47
Cast iron	9·74	25 to 85	9·54
Monel metal	38	—	27
Mild steel	25 to 30	26	20 to 24
Duralumin	31	—	—
Rolled brass	24·1	—	—
Cast brass	—	4·7	—

Example (1). *It is found that a bronze rod, $\frac{7}{8}''$ diameter, loaded gradually up to* 11 *tons, breaks just as that load is reached. . Calculate the strength of the bronze.*

Note.—If the load is applied in tension, compression, or shear, the strength will be the "tensile strength," "compressive strength," or "shear strength," respectively.

$$\text{Strength of the bronze} = \frac{11}{\frac{22}{28} \times \left(\frac{7}{8}\right)^2} = \frac{11 \times 28 \times 64}{22 \times 49}$$

$$= 18\tfrac{2}{7} \text{ tons per sq. in.}$$

Example (2). *A mild-steel plate has a shear strength of* 20 *tons per sq. in. If the plate is $\frac{1}{2}''$ thick, find the force needed to punch a $\frac{3}{4}''$ diameter hole through it.*

The area of metal parted is round the circumference of the hole, and $= \frac{22}{7} \times \frac{3}{4} \times \frac{1}{2}$ sq. in. in area.

It needs 20 tons to part a square inch.

$\therefore$ Force required $= \frac{22}{7} \times \frac{3}{4} \times \frac{1}{2} \times 20 = \mathbf{23\frac{4}{7}}$ **tons.**

Testing Machines

(1) Fig. 93 illustrates a rod of metal being tested in a tensile-testing machine. The load is applied gradually.

When a mild-steel rod is tested in tension up to breaking-point, it first elongates about 25 per cent. of its length before it breaks. At the break it necks. Mild steel always behaves in this manner, but other steels behave differently because of their inherent qualities; *e.g.* hard tool steel elongates very little and the neck is very slight; cast iron does not elongate or neck at all.

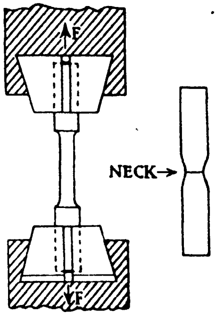


FIG. 93.

Example. *A mild-steel rod, $\frac{5}{8}''$ diameter, is subjected to gradually increasing loads up to $7\frac{1}{2}$ tons, when the specimen breaks. Calculate the intensity of stress as it breaks.*

$$\text{Intensity of stress} = \frac{\text{breaking load}}{\text{sectional area}}$$

$$= \frac{7\frac{1}{2}}{\frac{22}{28} \times \left(\frac{5}{8}\right)^2} = \frac{7\frac{1}{2} \times 28 \times 64}{22 \times 25}$$

$= \mathbf{24{\cdot}5}$ **tons per sq. in.**

(2) Fig. 94 illustrates a pin in double shear being tested for its shear strength.

P is the pin in double shear.

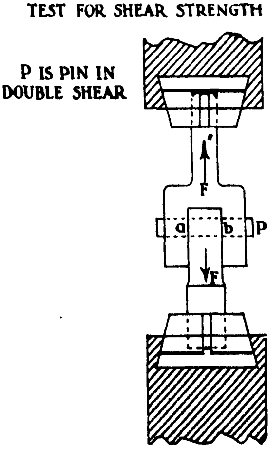


FIG. 94.

Example. *A $\frac{1}{4}''$ diameter steel pin shears under a load of 2 tons. Calculate the intensity of shear stress.*

The pin is in double shear; that is, two sections of the pin at *a* and *b* must be sheared.

$$\text{Shear stress} = \frac{2}{2 \times \frac{22}{28} \times \left(\frac{1}{4}\right)^2} = \frac{28 \times 16}{22}$$

$= \mathbf{20{\cdot}4}$ **tons per sq. in.**

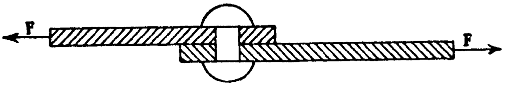


FIG. 95.

(3) Fig. 95 shows a rivet in shear stress; *i.e.* in single shear, as only one section of the rivet is resisting.

F = shearing load.

(4) Fig. 96 shows a link under a compressive load, putting the material in compressive stress.

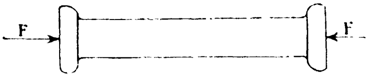


FIG. 96.

Hardness of Metals

Hardness is best defined as the resistance to penetration by other objects.

Two classes of test for hardness have been devised :

(1) **Indentation Tests,** and

(2) **Scoring or Scratching Tests.**

Indentation tests have been designed with punches or indenting tools of various shapes, and the depth, volume, or super-area of the indentation made by a given pressure is measured and the amount gives an indication of the hardness of the surface of the metal which is being tested.

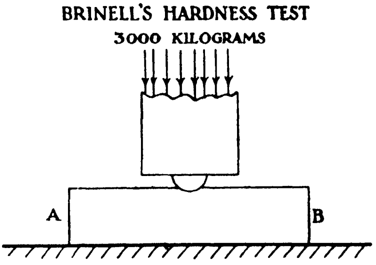


FIG. 97.

The following particulars relate to **Brinell's Hardness Test.**

In Fig. 97 AB is a piece of metal whose hardness of surface is required to be known.

The plunger above the metal has a vertical pressure on it of 3000 Kilograms, and the spherical surface at the bottom—10 millimetres diameter—makes an impression on the surface of the metal.

The diameter of this impression is measured, and **Brinell's Hardness Number** is calculated from the formula :

$$\textbf{Brinell hardness number} = \frac{\textbf{total pressure}}{\textbf{curved area of depression}}.$$

As stated previously, the "total pressure" used is 3000 Kilograms, and the curved area of the indentation is obtained from the formula :

$$2\pi r\left(r - \sqrt{r^2 - \frac{d^2}{4}}\right),$$

where r is the radius of the spherical ball at the bottom of the plunger and d is the diameter of the impression which is measured by a scale across the lens of a microscope, specially used for the purpose.

Brinell hardness number $= \dfrac{3000}{10\pi\left(5 - \sqrt{25 - \dfrac{d^2}{4}}\right)}$.

Example. *Let the diameter of the indentation "d" equal* 5 *millimetres, then the curved area of the depression*

$$= 31·42\left(5 - \sqrt{25 - \frac{25}{4}}\right)$$

$$= 31·42\left(5 - \sqrt{\frac{75}{4}}\right) = 31·41(5 - 4·33)$$

$$= \mathbf{31·42 \times 0·67} \textbf{ square millimetres.}$$

Hence Brinell's hardness number $= \dfrac{3000}{31·42 \times 0·67}$

$$= \mathbf{143}.$$

This result is shown in the table below, where $d = 5$ mm.

The Brinell Hardness Test
and corresponding approximate Tensile Strength
(Maximum Stress)
Size of Ball 10 mm. dia. Pressure 3,000 Kgs.

Dia. of Ball Impression.	*Brinell Numeral.*	*Tons per □ in.*	*Kilos. per □ mm.*	*Dia. of Ball Impression.*	*Brinell Numeral.*	*Tons per □ in.*	*Kilos. per □ mm.*
2·3	713	170	267	3·75	262	58	91
2·4	652	156	245	3·8	255	56	88
2·5	600	135	208	3·85	248	55	86
2·6	555	125	192	3·9	241	54	85
2·65	532	122	179	3·95	235	52	82
2·7	512	120	176	4·0	228	51	80
2·75	495	116	170	4·1	217	49	77
2·8	477	112	165	4·2	207	47	74
2·85	460	107	160	4·3	196	44	69
2·9	444	102	155	4·4	187	42	66
2·95	432	98	150	4·5	179	40	63
3·0	418	94	145	4·6	170	39	61
3·05	402	90	139	4·7	163	37	58
3·1	389	87	134	4·8	156	36	57
3·15	375	84	130	4·9	149	34	54
3·2	364	81	126	5·0	143	33	52
3·25	351	77	122	5·1	137	31	49
3·3	340	74	118	5·2	131	30	47
3·35	332	72	114	5·3	126	29	46
3·4	321	70	111	5·4	121	28	44
3·45	311	68	107	5·5	116	27	43
3·5	302	66	104	5·6	112	26	41
3·55	293	64	101	5·7	107	25	39
3·6	286	63	99	5·8	103	24	38
3·65	277	61	96	5·9	99	22	35
3·7	269	60	93	6·0	95	21	33

By the courtesy of Messrs. Crofts.

Elasticity

Any material which will change in shape when loaded but will return to its original form when the load is removed is said to be **elastic.** Thus, rubber is elastic. Springs are made of some metal or other, and these metals are elastic. A plank of timber bends when loaded and springs back when unloaded; thus, timber is elastic. **All gases are very elastic**; it is their elasticity which enables engines to work.

Such materials as lead, putty, and clay are not elastic.

The amount of elasticity can be altered by various operations.

In metals the **elasticity can be destroyed** (1) by raising the temperature, (2) by loading too much. The student will observe that if a plate of steel is made red hot, it bends easily and will not return of its own accord. Also he may pull a spiral spring out so that it will not return, that is, he loads it too highly.

With rubber, it is said to *perish* through lying idle for a long time; that is, its elasticity disappears.

Elastic Materials.	*Materials not Elastic.*
Rubber.	Putty.
Timber.	Clay.
Steel.	Tallow.
Brass.	Pulp.
Bronze.	
All gases.	

The elasticity of the metals noted in the table is very evident in the case of springs made of the metal. However, the pupil must understand that the materials stated as elastic are only so under normal or ordinary conditions. Under abnormal affairs or according to production requirements, the elasticity of any material may be destroyed.

Experiment on the Elasticity of a Copper Wire

A copper wire, 12′ 3″ long and ·036″ diameter, fixed at the top to a beam and loaded at the bottom with increasing loads of *w* lb. weight, was noted to stretch and the loads and extensions are noted in the following table. The loads and extensions are plotted in the graph below (Fig. 98).

Load lb.	2	4	6	8	10	12	14	16	18	20	22	24
Extension in.	·02	·041	·06	·082	·101	·121	·142	·161	·180	·201	·51	Wire broke.

From the graph it will be seen that the metal remained elastic up to a certain point in the loading. This point is marked on the graph as the "elastic limit," that is, the point in the loading at which the metal crosses over from being elastic to being plastic. Before this point is reached, it will be noted that a straight line can be drawn through the plotted points and therefore from this particular it follows that the extension is proportional to the amount of the load. After this point in the loading the metal becomes plastic and the particles or crystals of the metal move easily under each additional load. The wire broke when the load reached 24 lb.

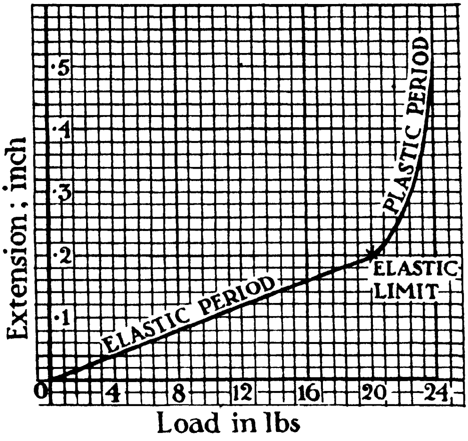


FIG. 98.

The intensity of breaking stress is given by the fraction, in tons per square inch:

$$\frac{24}{\frac{22}{28} \times (\cdot 036)^2 \times 2240}$$
$$= 10\cdot 52 \text{ tons per square inch.}$$

This is called "the strength of the copper in tension" or the "**tensile strength.**"

At the elastic limit the

$$\text{intensity of stress} = \frac{20}{\frac{22}{28} \times \cdot 036^2 \times 2240} \text{ tons per square inch}$$
$$= \mathbf{8.77} \textbf{ tons} \text{ per square inch.}$$

Factor of Safety.

The factor of safety is a number by which the strength of a material is divided in order to get the maximum normal working stress allowed to come into the material under the ordinary working load.

$$\textbf{Working stress allowed per sq. in.} = \frac{\textbf{strength of material per sq. in.}}{\textbf{factor of safety}}.$$

Example (1). *The strength of mild steel is* 28 *tons per sq. in. Take a factor of safety* 4 *and calculate the intensity of working stress allowed.*

$$\text{Working stress allowed} = \frac{28}{4} = \mathbf{7} \textbf{ tons} \text{ per sq. in.}$$

Example (2). *A link of steel is* 2" *wide and* ¼" *thick. If the tensile strength of the steel is* 28 *tons per sq. in., take a factor of safety* 4 *and calculate the greatest pull which may come on the link.*

$$\text{Working stress allowed} = \frac{28}{4} = 7 \text{ tons per sq. in.}$$

$$\text{Area of section} = 2 \times \tfrac{1}{4} = \tfrac{1}{2} \text{ sq. in.}$$

$$\text{Greatest pull} = 7 \times \tfrac{1}{2} = \mathbf{3\tfrac{1}{2}\ tons.}$$

Example (3). *A brass rod has to support a load of* 18 *tons. The rod is square in section and the tensile strength of the brass is* 24 *tons per square inch. Take a factor of safety* 6 *and calculate the edge of the square section of the rod.*

$$\text{Working stress} = \frac{24}{6} = 4 \text{ tons per sq. in.}$$

$$\text{Area of section} = \frac{\text{load}}{\text{working stress}} = \frac{18}{4} = 4\tfrac{1}{2} \text{ sq. in.}$$

$$\sqrt{4\tfrac{1}{2}} = \text{length of edge of the square} = \mathbf{2{\cdot}14''}.$$

Working Metals to Shape

In working an object made of metal into the required shape it is obvious that the working tool must overcome the strength of the metal, for the tool is tearing particles of metal from their neighbours. This process is easily appreciated in the operations of turning with the lathe; planing on the planing machine; and also with the milling tools, etc.

Similarly, in the case of press tool operations, although the metal is not torn asunder in this class of work, yet the particles are made to flow as in a very viscous fluid. Obviously, when metal is being pressed into shape, the elastic limit of the metal has been passed and the object takes a permanent form.

A very comprehensive treatise on the *Machinability of Metals* was read before the Institution of Mechanical Engineers by E. G. Herbert, B.Sc., and the reader is advised to read the paper, which is published in the *Transactions* of the Institution.

Example. *A disc of* 1¾" *diameter is pressed out of a sheet of mild steel* ¼" *thick. The shearing strength of the steel is* 20 *tons per sq. in. Calculate the force required to shear the disc out.*

The area of the metal sheared is round the edge of the disc:

$$\text{Area} = \frac{22}{7} \times 1\tfrac{3}{4} \times \tfrac{1}{4} \text{ sq. in.}$$

$$= \frac{\overset{11}{\cancel{22}}}{\cancel{7}} \times \frac{\cancel{7}}{\underset{2}{\cancel{4}}} \times \frac{1}{4} = \frac{11}{8} \text{ sq. in.}$$

$$\text{Shearing force required} = \overset{5}{\cancel{20}} \times \frac{11}{\underset{2}{\cancel{8}}} \text{ tons.}$$

$$= 27\tfrac{1}{2} \textbf{ tons.}$$

In the above example the word " **force** " is used, but the word " load " is just as applicable in this case. The word " force " may be applied to a variety of cases of loading, such as (1) steam pressure, (2) cutting force of tools, (3) force in belts or ropes, etc., but in every case, it is some form of loading applied to an object and is measured by the same units ; *i.e.* pounds, tons, grams and Kilograms, poundals, etc.

EXERCISE 14

(1) A tie bar 2" wide and 1¼" thick is subject to a direct pull of 7 tons. Calculate the intensity of tensile stress.

(2) If the bar in question (1) has a hole ⅝" diameter bored through it with the centre of the hole in the middle of the 1¼" face, what is now the intensity of stress at the section through the diameter of the hole ?

(3) A square hole, ½" edge, is being pressed through a brass plate ¼" thick. If the shear strength of the brass is 12 tons per sq. in. what force is needed to push the punch through ?

(4) A rivet is ½" diameter and is in single shear. The shearing strength of the metal is 20 tons per sq. in. (*a*) What load will shear the rivet ? (*b*) If the factor of safety for the rivet is 5, what is the greatest working load allowed on the rivet ?

(5) Mild steel has a shearing strength of 20 tons per sq. in. What force is needed to press a ¾" diameter hole in a steel plate ⅝" thick ?

(6) In the case of the punch in question (5), what is the intensity of stress in the punch ? State the kind of stress in the punch.

(7) A test bar of mild steel ⅝" diameter is subject to gradually increasing loads. At 6½ tons load it was found the elastic limit was reached. Calculate the intensity of stress at the elastic limit.

(8) A long wire stretched ·55" under a load of 50 lb. What would be the extension under a load of 30 lb. ? (Assume that the elastic limit is not passed.)

(9) A rod of metal, supported at the ends, deflects ½" in the middle when a load of 500 lb. is applied at the middle point. What will be the deflection for a load of 200 lb. ?

(10) The following results were noted when doing a tensile test on a rod of duralumin. The original length of the specimen was 8" and the diameter ⅛".

Load lb.	50	100	150	200	250	300	400	500	550	600	650
Extension in.	·001	·002	·0031	·004	·0049	·006	·008	·01	·011	·012	·05

On squared paper, plot a graph of the particulars from the experiment.

Mark on the graph the elastic period ; also the elastic limit.

What is the intensity of stress at the elastic limit ?

(11) (*a*) As mild steel elongates about 25 to 30 per cent. of its original length before it breaks under tension, is there any connection between this fact and the malleability or ductility of the metal ?

(*b*) In a shear test on a ¼″ diameter mild-steel rivet in double shear, it was found that a load of 5160 lb., gradually applied, sheared the rivet. From these figures calculate the shearing strength of mild steel (1) in pounds per square inch, and (2) in tons per square inch.

(12) As in the last question, a test was made on a ¼″ diameter brass rivet, in double shear. It was found that a load of 1½ tons, gradually applied, sheared the rivet. Calculate the shear strength of brass in tons per square inch.

(13) In a mild-steel rivet in single shear a load of 1¾ tons is applied. The rivet is ⅝″ diameter. Calculate the intensity of shear stress in the rivet.

(14) The shear strength of brass is found to be 15·27 tons per sq. in. Calculate the force required to shear a ¾″ diameter hole in a ½″ thick plate.

(15) In the last question, find the compressive stress in the punch.

(16) The shear strength of mild steel is 23·7 tons per sq. in. Take a factor of safety 4 and calculate the greatest load which may come on a 1″ diameter rivet in double shear.

(17) The gudgeon pin for a steam-engine cross-head is turned to be 2″ diameter. Take the shear strength of the metal to be 20 tons per sq. in. and calculate the shear resistance when the pin is in double shear. (Adopt the practical rule that double shear gives 1¾ the strength of single shear.)

(18) A steel rod, circular in section and ½″ diameter, is subject to a direct pull of 2½ tons. Calculate the intensity of tensile stress.

(19) A tensile test was made on a metal wire 0·05″ in diameter and 84″ long. The following loads and extensions were recorded :

Load lb.	0	5	10	15	20	25	30
Extension in.	0	·016	·031	·044	·061	·142	·41

Plot a load-extension graph on squared paper.

Mark on it the elastic limit.

Calculate the intensity of stress at the elastic limit.

(20) The following results were obtained in testing a certain wire for extension. Obtain the extension for the various loads given and plot a load-extension graph. Explain what is indicated by the graph within the limits of the experiment.

Load.	*Reading of Vernier.*	*Extension (by difference).*
lb.	in.	
0	·01	
10	·09	
20	·165	
25	·21	
30	·25	
35	·285	

(21) (*a*) Rivets, bolts, pins, and cotters are sometimes in "single shear" or "double shear." Give two sketches illustrating (i) single shear, and (ii) double shear.

(*b*) A shackle pin is 1½ sq. in. in sectional area, and is in double shear. It supports a load of 12 tons. Calculate the intensity of shear stress.

(22) (*a*) What is meant by "elasticity" in regard to engineering materials?

(*b*) State three materials which are elastic, and three which are not elastic.

(*c*) When links of machines are loaded, they are said to be in a "state of stress." Define what this means.

(23) When an object or machine link is loaded, the material of which it is made is said to be placed in a state of "stress."

(*a*) Describe what is meant by "the plastic state of metals."

(*b*) How do engineers usually state the "intensity of stress"?

(*c*) A machine link ½" square section has a pull of 6248 lb. at its ends. Calculate the intensity of stress in the material of the link. State the nature of the stress.

CHAPTER XV

LEVERS, MOMENTS OF FORCES

In mechanics, the effect of a force is usually increased by applying the force on an object by means of a lever. Of course, a huge variety of forms of lever are met with in mechanisms and tools ranging from the ordinary pinch-bar to tooth wheels and pulleys. Examples of hand tools which work on the principle of the lever are: (1) spanners, (2) pincers, (3) screwdrivers, (4) stocks for dies, (5) machine handles for adjustments, etc., etc.

Moment of a Force

This is a term commonly used in the mechanics of levers.

Definition. *The moment of a force is the product of the force and the vertical distance from the line of action of the force to the centre about which the lever is acting.*

Illustration of a Moment (Fig. 99)

Let the straight line and the arrow-head represent the force of F lb., and O the point about which the lever x is turning. Then F lb. multiplied by x inches gives the moment of F about O in pounds/inches.

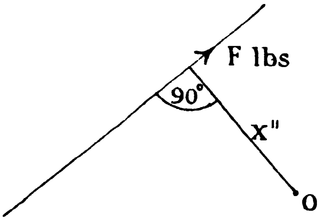


Fig. 99.

The point O is called the **fulcrum,** although in workshop language it may be called by other names, such as "the bearing," or "the pivot," or the "knife edge."

In the illustration of a moment, the force F lb. is shown to turn the lever in a clockwise direction—moments turning in this direction are called **clockwise moments,** while moments turning in the opposite direction are called **anti-clockwise moments.**

In any case of leverage, a clockwise moment is opposed by an anti-clockwise moment and that to balance, these two opposing moments are equal.

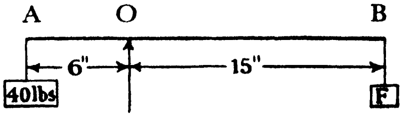


Fig. 100.

In Fig. 100, AB represents a dimensioned lever, with fulcrum at O and a weight of 40 lb. acting at A. What weight or force F at B will balance this?

Anti-clockwise moment = 40 × 6.
Clockwise moment = F × 15.
Clockwise moment = anti-clockwise moment for balance.

$$\therefore F \times 15 = 40 \times 6. \quad \therefore F = \frac{40 \times 6}{15} = 16 \text{ lb.}$$

PS.—The weight of the lever is neglected in this example.

Example. *Fig.* 101 *illustrates a tap wrench. A force of* 7 *lb. is applied by the hands, the centre of each hand being* 8″ *from the centre of the tap. Calculate the total turning moment.*

Both forces give a clockwise moment of 7 × 8 lb./in. each. Therefore the total clockwise moment = 2 × 7 × 8 = 112 lb./in. The anti-clockwise moment is caused by the resistance of the tap round its centre line, which is the fulcrum in this case.

The above illustration on the tap wrench is a good picture of a **couple** as known in mechanics. A **couple** is defined in mechanics as *two equal forces acting along parallel lines, but opposite in direction.*

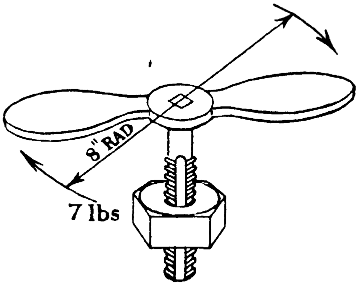


FIG. 101.

Orders of Levers (Fig. 102)

Levers are said to be of different orders according to the position of the fulcrum in relation to the effort and the load.

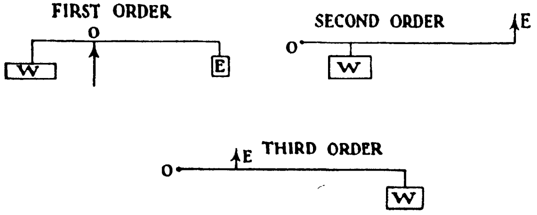


FIG. 102.

First Order. Fulcrum O between the load W and effort E.

Second Order. Fulcrum at one end, effort E at the other end, load W between O and E.

Third Order. Fulcrum at one end, load W at the other end, and effort E between O and W.

All three orders are commonly found in hand tools and in machine tools.

Example. The tap wrench (Fig. 101) above is an example of the first order.

The pincers shown in Fig. 103 are a pair of levers working on the fulcrum at the pin O. *Both levers are of the first order.*

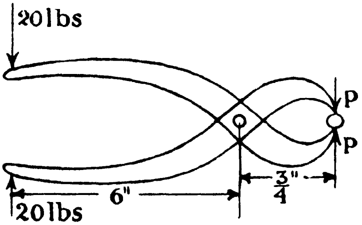


FIG. 103.

Let P equal the pressure at the grip. Taking moments about O,

$$P \times \tfrac{3}{4} = 20 \times 6$$

$$P = \frac{20 \times 6}{\frac{3}{4}} = 20 \times 6 \times \frac{4}{3}$$

$$P = 160 \text{ lb.}$$

Pinchbar (Fig. 104)

A man presses down with a force of 120 *lb. If* $x = 48''$ *and* $y = 3''$, *calculate the upward lift on the wheel* (Fig. 104).

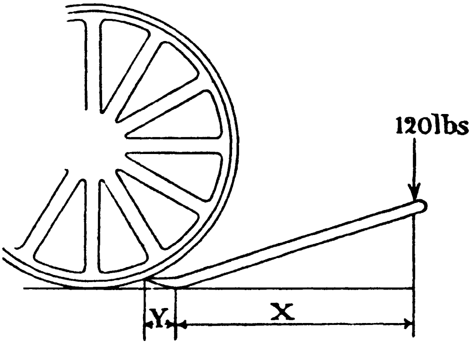


FIG. 104.

Let L = the upward lift.
Clockwise moment = 120 × 48.
Anti-clockwise moment = 3L.

$$\therefore 3L = 48 \times 120. \quad L = \frac{120 \times 48}{3} = 1920 \text{ lb.}$$

Hand Shearing Machine (Fig. 105). The lever is of the second order.

Moment of F lb. about the fulcrum = F × L.

Let F = 20 lb. and L = 24".

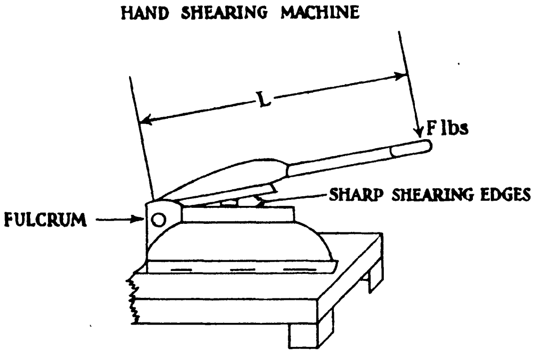


FIG. 105.

If the distance of the job from the fulcrum equals 3″, let P be the shearing force on it.

Then $P \times 3 = 20 \times 24$. $\therefore P = 160$ lb.

The third order of levers is shown in the sketch of the safety valve (Fig. 106).

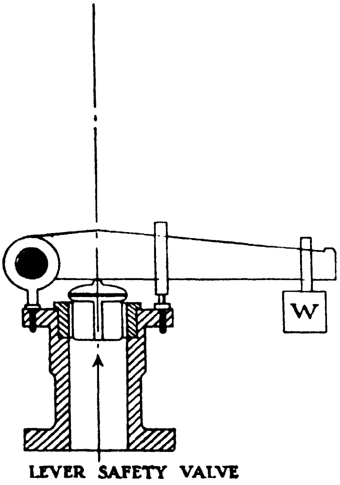


FIG. 106.

O is the fulcrum, E is the effort near to the fulcrum, and W is the weight at the other end of the lever.

Example. Let the distance of the line of action of W be 24″ and the distance of E be $1\frac{1}{2}$″.

E is the upward pressure of the steam on the valve above, which amount the steam must blow out.

Let $W = 56$ lb.
Then 56×24 = anti-clockwise moment,
and $E \times 1\frac{1}{2}$ = clockwise moment,
about fulcrum O.

$$\therefore E \times 1\tfrac{1}{2} = 56 \times 24, \textit{ i.e. } E = \frac{56 \times 24}{1\frac{1}{2}}.$$

$$E = 56 \times 24 \times \tfrac{2}{3} = \mathbf{896\ lb.}$$

When the steam pressure gets slightly above this amount, it blows out and the pressure gets no higher.

Systems of Levers

Fig. 107 shows a system of levers used in a small testing machine. F is the test specimen in tension. F is held in chucks at H and K. At K, F is rigidly held at rest. H tends to move upwards. The lever DE moves round a fulcrum at E and the lever AC moves round a fulcrum at B. CD is a link on pin joints at C and D.

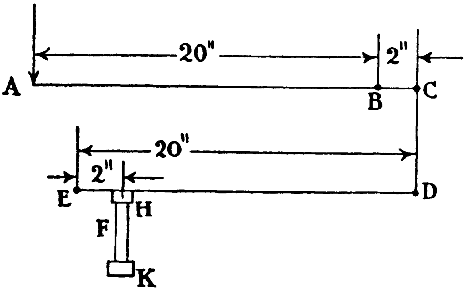


FIG. 107.

Let a vertical force of 40 lb. be exerted at A.

Then the upward force on link CD $= 40 \times \frac{20}{2} = 400$ lb.

The upward force on the specimen F is given by :

$$F = 400 \times \frac{20}{2} = \mathbf{4000\ lb.}$$

EXERCISE 15

(1) The lengths between the centres of the nuts for the spanners (Fig. 108) shown in the figure below are marked on them. Assume that a man pulls with a force of 20 lb. at a point 2" distant from one end, and calculate the turning moment in each case.

(2) Make six sketches showing hand tools which work by the principle of the lever. Mark the fulcrum in each case.

(3) By using an ordinary ruler over a fulcrum, show how by having a weight of one pound you could weigh any quantity from 1 lb. to 6 lb., including ounces. The ruler must be divided into sixteenths of an inch.

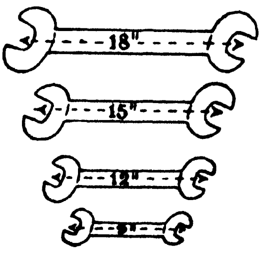


FIG. 108.

(4) A belt gives a force of 150 lb. to the circumference of a driven pulley, 18″ diameter. What turning moment is given to the driven shaft?

(5) A man is tapping a hole with stocks and dies. The length of the stocks from hand to hand is 30″. The force of each hand is 15 lb. What is the turning moment? In this case show that a "couple" is acting.

(6) A lad is blowing a bellows at a small forge. From his hand to the fulcrum is 4′, and from the fulcrum to the pressure pin is 3″. If the lad pulls with a force of 18 lb., what total pressure is on the air in the bellows?

(7) In a lever safety valve, the valve spindle is 1½″ from the fulcrum. A load of 56 lb. is hung from a point on the lever 21″ from the fulcrum. Find the total steam pressure on the valve when the steam "blows off."

(8) (*a*) In driving a motor-car, show that a man usually applies a couple to the steering-wheel.

(*b*) If each of his hands exerts a force of 4 lb. and the effective diameter of the wheel is 15″, what twisting moment does he exert?

(9) A lad pulls at the end of a lever with a force equal to a weight of 90 lb. The effective length of the lever is 8′ 4″. Calculate the moment of his force.

(10) In a pair of hand shears the distance of the hand from the pivot is 24″, while the distance of the job from the pivot is 1″. If the man exerts a pressure of 30 lb., what pressure will be exerted on the job?

(11) A rod crosses two supports 6′ between them. A load of 240 lb. hangs from a point 2′ from one support. Find the load on each support.

(12) The lathe chuck exerts a twisting moment of 4000 lb./in. on a piece of work 4″ diameter. Calculate the cutting force at the tool point.

(13) A pair of blacksmith's tongs has the following dimensions: from the pressure on the metal to the pin or fulcrum 2″; from the grip of the hand to the fulcrum 22″. If the pressure of the grip is 8 lb., find the pressure on the metal.

(14) The cutting force of a milling cutter is 700 lb. and the radius of the cutter is 2½″. Calculate the twisting moment or torque transmitted by the spindle of the machine.

(15) In a lever safety valve for a boiler, from the fulcrum to the centre of the valve is 2½″. From the point of suspension of a 56-lb. weight to the fulcrum is 15″. What will be the total steam pressure on the valve when the steam blows off?

(16) The pressure on the teeth of a wheel when a tooth is fully in gear is 950 lb. The pitch diameter of the wheel is 14″. Calculate the twisting moment transmitted by the wheel. (Note.—Only one tooth is fully in gear at one time.)

(17) A lathe tool exerts a cutting force of 350 lb. on a piece of work 4″ diameter. The work is driven at the headstock by a carrier or face-plate pin. From the centre of the face-plate pin to the centre of the work is 7″. Calculate the pressure exerted by the pin.

(18) A man is drilling a hole with a ratchet brace. The distance between the centre line of the drill and the pull of the hand is 15″. The average effective cutting pressure of the drill on the work is at a point $\frac{3}{8}$″ radius. The man pulls with a force of 12 lb. What is the cutting pressure on the work?

(19) In the case of a leg vice, the distance from the fulcrum to the pressure point on the jaws is 27″. The distance from the fulcrum to the centre of the screw is 18″. When the screw exerts a pressure of 1 ton, what will be the pressure between the jaws on the work?

(20) In wood-turning, the cutting force of 25 lb. exerted by the gouge is 2″ from the tool rest. The distance of the hand pressure from the tool rest is 10″. Calculate the downward pressure of the hand.

CHAPTER XVI

TRANSMISSION OF MOTION

Speed or Velocity

Speed means the rate of change of position, and **velocity is speed with the direction** stated as well. Speed is called a **scalar quantity,** while velocity is called a **vector quantity.** The engineer may speak of the cutting speed of a machine without denoting the direction, and thus this statement of speed is a **scalar quantity.** On the other hand, an engineer may speak of the speed of 200 miles an hour of an aeroplane in a direction of forty degrees with the true North—this is a **vector quantity.**

Fig. 109 illustrates the velocity of the aeroplane as stated, and this is the manner in which engineers denote vectors; *i.e.* with straight lines and arrow-heads to show the direction, and with the quantity stated against the line.

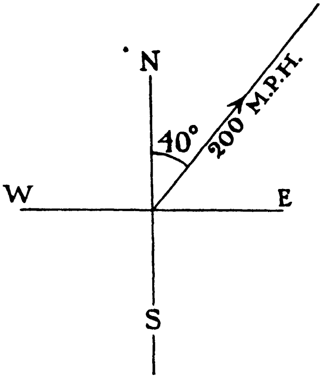


FIG. 109.

Example (1). *A planing-machine table makes six cutting strokes of 8′ each in length per minute. Find the cutting speed, assuming that the cutting takes place only on the outward stroke and that the return speed is the same as the cutting speed.*

Under these conditions, the cutting is done in half a minute.

$$\text{Distance of work cut} = 6 \times 8'.$$

$$\text{Time of cutting} = \tfrac{1}{2}\text{ min.}$$

$$\text{Cutting speed} = \frac{\text{distance}}{\text{time}} = \frac{6 \times 8}{\frac{1}{2}}$$

$$= 6 \times 8 \times 2 = \mathbf{96' \text{ per min.}}$$

Example (2). *A shaft is being turned in a lathe. At a certain instant the diameter of the work is* 10½″. *If the revolutions per minute number* 180, *find the cutting speed in feet per minute.*

$$\text{Cutting speed} = \frac{22}{7} \times \frac{10\frac{1}{2}}{12} \times 180$$

$$= \mathbf{495' \text{ per min.}}$$

Example (3). *In a milling cutter, the effective diameter of the cutter is 6", and it makes 90 revolutions a minute. Calculate the cutting speed.*

$$\text{Cutting speed} = \frac{22}{7} \times \frac{6}{12} \times 90 \text{ ft. per min.}$$
$$= 141\tfrac{3}{7}' \text{ per min.}$$

Example (4). *A belt pulley,* 21" *diameter, makes* 140 *revolutions per minute. Calculate the belt speed in feet per minute.*

$$\text{Belt speed} = \frac{\overset{11}{\cancel{22}}}{\cancel{7}} \times \frac{\overset{\cancel{3}}{\cancel{21}}}{\underset{\underset{\cancel{2}}{\cancel{4}}}{\cancel{12}}} \times \overset{70}{\cancel{140}} = 770' \text{ per min.}$$

Example (5). *A cyclist is moving along a level track at* 10 *miles an hour. If the wheels are* 24" *in diameter, calculate how many revolutions per minute the wheels make.*

$$\text{Circumference of wheels} = \frac{22}{7} \times \frac{24}{12} = \frac{44}{7} \text{ ft.}$$

$$\text{Distance cycle goes in a minute} = \frac{10 \times 1760 \times 3}{60} = 880 \text{ ft.}$$

$$\text{Revolutions of wheels per min.} = \frac{880}{\frac{44}{7}} = 880 \times \frac{7}{44} = \mathbf{140.}$$

Change of Speed

Pulleys of different diameters are used for the purpose of changing the speed of a shaft in revolutions per minute.

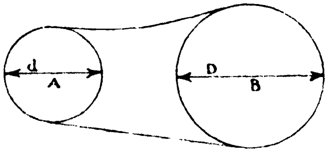


FIG. 110.

In Fig. 110, A is the driving pulley and B is the driven one. Every part of the belt has the same speed, and it passes over both pulley rims.

$$\therefore \pi dn = \pi DN$$

where n and N are the revolutions of the small and large pulleys respectively per minute.

From the equation cancel π out, then

$$dn = DN$$

i.e. the diameter of the pulley multiplied by its number of revolutions per minute is the same value for both pulleys.

Example. *Let the diameter of pulley* A *be* $1\frac{1}{2}'$, *and make* 400 *revolutions per minute. If pulley* B *is required to make* 150 *revolutions a minute, then from the formula :*

$$1\tfrac{1}{2} \times 400 = D \times 150,$$

$$i.e.\ \frac{1\frac{1}{2} \times 400}{150} = D. \quad \therefore D = 4'.$$

This is the size of pulley required.

Crossed belts (Fig. 111) are used to give a speed in the reverse direction. The calculation is the same, whether the belts are open or crossed.

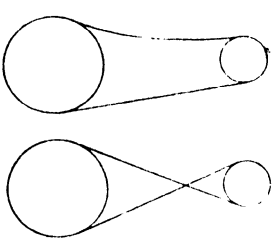


FIG. 111.

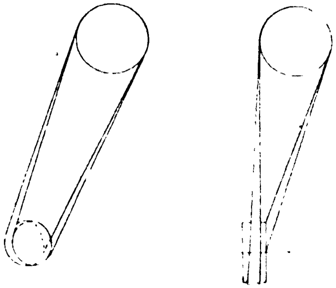


FIG. 112.

The position of the belt drives (Fig. 112) does not alter the calculation either.

Cone Pulleys

Cone pulleys are used for the purpose of varying the speed of the machine spindle, although the number of revolutions per minute of the line shaft or counter-shaft remains the same. Fig. 113 shows the arrangement of a cone pulley.

To increase the number of revolutions per minute of the spindle above those of the shaft, the belt should be on a shaft pulley larger in diameter than the one on the spindle.

To reduce the speed of the spindle below that of the shaft in revolutions per minute, the belt should be on a shaft pulley smaller in diameter than the one on the spindle.

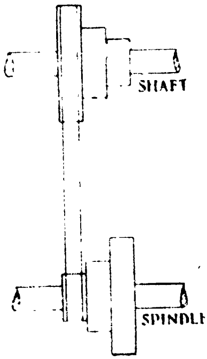


FIG. 113.

Example (1). *A cone pulley on a counter-shaft for belts has three steps—the largest being* 12″ *diameter and the smallest step* 4″ *diameter—the cone pulley on the machine being a similar one to that on the line shaft. If the counter-shaft makes* 120 *revolutions per minute, find the range of speed the machine spindle has.*

Working from the formula :

Revolutions per minute × *diameter* is the same for both pulleys, let n be the revolutions per minute of the machine spindle when the belt is on the large step of the counter-shaft cone. In this case the belt will be on the small step of the machine-spindle cone pulley.

$$\text{Then } n \times 4 = 120 \times 12$$

$$n = \frac{120 \times 12}{4} = 360.$$

Now let the belt be on the small step of the counter-shaft cone pulley and on the big step of the spindle cone pulley.

Let N be the revolutions per minute of the machine spindle in this case.

$$\text{Then } N \times 12 = 120 \times 4$$

$$N = \frac{120 \times 4}{12} = 40.$$

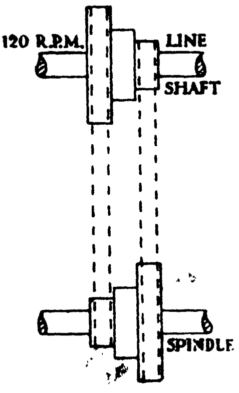


FIG. 114.

Thus, the range of speed for the machine spindle for 120 revolutions per minute of the counter-shaft is 40 → 360.

Example (2). *A line shaft revolves at* 120 *revolutions a minute and has a cone pulley the bottom step of which is* 6″ *diameter and the top step is* 15″ *diameter. If the cone pulley on the machine spindle is the same, find the range of speed of the machine spindle.*

Revolutions × diameter is the same for both machines.

First, let the belt (Fig. 114) be on the smallest step of the line-shaft pulley.

Let n = revolutions per minute of the lathe spindle.

120×6 = revolutions of spindle $\times 15 = n \times 15$.

$$\text{Let } n = \frac{\overset{24}{\cancel{120}} \times \overset{2}{\cancel{6}}}{\underset{\cancel{5}}{\cancel{15}}}$$

= **48 revolutions.**

Now let the belt be on the big step of the line-shaft pulley.

Let N = revolutions of the spindle.

Revolutions × diameter = revolutions × diameter.

$$120 \times 15 = \text{revolutions} \times 6$$

$$= \frac{120 \times 15}{6} = N$$

$$= 300 = N.$$

Hence, if the intermediate pulleys are the same diameter, then 120, 48, and 300 are the three speeds of the machine spindle.

Gear Drives

Similar calculations to the ones for belt and rope drives can be made for tooth-wheel drives.

In Fig. 115 let A be the driver and B be the driven wheels respectively.

The student should note particularly that only one tooth on each wheel is fully in gear at one time. Hence for every tooth on B which gears, there must be a tooth on wheel A to gear with it.

Let wheel A have 22 teeth and make 400 revolutions a minute.

Then 400 × 22 is the number of teeth gearing per minute on both wheel A and wheel B.

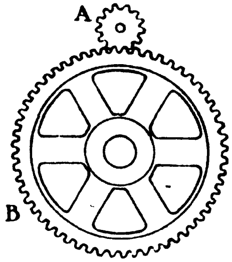


FIG. 115.

Let wheel B have 110 teeth, then if N is the number of revolutions per minute, 110N is the number of teeth gearing per minute on wheel B.

$$\therefore 110N = 400 \times 22, \quad \textit{i.e.}\ N = \frac{40\cancel{0} \times \overset{2}{\cancel{22}}}{\cancel{110}}.$$

$$N = 80.$$

Worm and Worm Wheel

In the worm and worm wheel, generally, the worm makes a complete revolution for an advance of one tooth on the worm wheel. Thus, if the wheel has *n* teeth, the velocity ratio between the worm and wheel is *n* : 1. In Fig. 116 it is 30 to 1

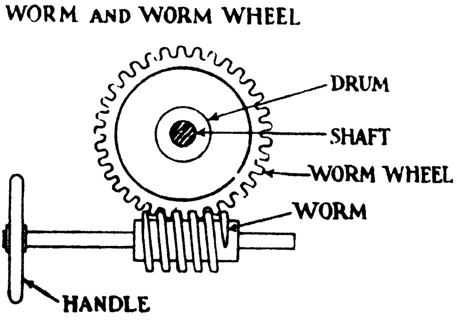


FIG. 116.

Gear of a Bicycle

The ordinary gear of a bicycle :

$$\textbf{Gear} = \frac{\textbf{number of teeth on crank wheel}}{\textbf{number of teeth on hub wheel}} \times \textbf{diameter of back wheel.}$$

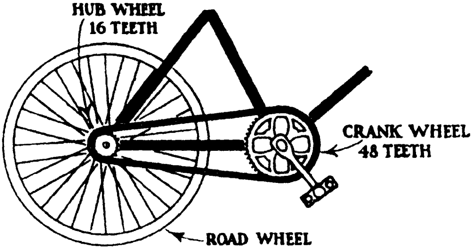


FIG. 117.

In Fig. 117, let the diameter of the back wheel be 26″. The crank wheel has 48 teeth and the hub wheel 16 teeth.

$$\text{Gear} = \frac{48}{16} \times 26 = 78.$$

If the bicycle is fitted with a three-speed gear the above result will have to be multiplied by certain factors depending on the design of the gear.

The gear of a bicycle is the equivalent diameter of the back wheel of the bicycle for one revolution of the crank, assuming the crank fastened to the hub of the back wheel.

Example. The gear in the above case is 78.

The bicycle moves forward $\frac{22}{7} \times 78''$ for each revolution of the crank, *i.e.* for each revolution of the crank the bicycle moves forward $245\frac{1}{7}''$.

The Lathe Headstock

Fig. 118 shows a gearing at the lathe headstock. If wheel A has 20 teeth and wheel B has 100 teeth, then the lead screw will make one-fifth the number of revolutions of the lathe spindle.

If the revolutions per minute of the mandrel or lathe spindle is 150, then the lead screw will make 30 revolutions per minute.

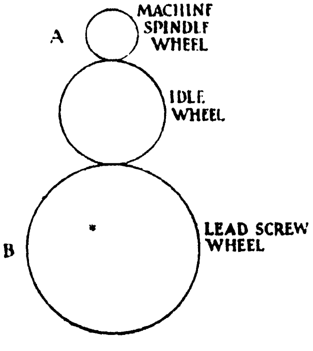


FIG. 118.

Compound Train at the Lathe Headstock

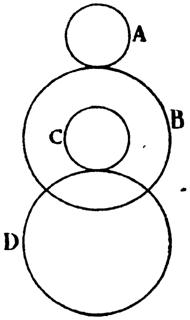


FIG. 119.

Wheels B and C (Fig. 119) are keyed on the same sleeve and make the same number of revolutions as they revolve together.

Let wheel A (Fig. 119) be on the mandrel or spindle and have 20 teeth. Let wheel B have 80 teeth. Then wheels B and C will make quarter the revolutions of wheel A.

Now let wheel C have 30 teeth and wheel D 90. Then wheel D will make one-third the revolutions of wheel C.

Hence wheel D will make $\frac{1}{3}$ of $\frac{1}{4}$ of the revolutions of wheel A.

Therefore, if wheel A makes 180 revolutions a minute, then wheel D will make $\frac{180}{12} = 15$ revolutions a minute.

A and C are said to be drivers, and wheels B and D are driven wheels.

Then, generally, velocity ratio between wheel A and wheel D is given by the formula :

$$\textbf{Velocity ratio} = \frac{\textbf{product of numbers of teeth on driven wheels}}{\textbf{product of numbers of teeth on drivers}}.$$

In the latter case :

$$\text{Velocity ratio} = \frac{90 \times 80}{30 \times 20} = 12.$$

If wheel D is on the lead screw, then the mandrel makes twelve times the number of revolutions that the lead screw makes in the same time.

Screw-cutting

The train of wheels for screw-cutting will now be considered.

The fraction to start with is : $\frac{\text{Revolutions per minute of lead screw}}{\text{Revolutions per minute of the work}}$.

i.e. $\frac{\textbf{Threads per inch on the lead screw}}{\textbf{Threads per inch on screw to be cut}}$.

Let us require to cut a thread 15 to the inch length, by a lead screw $\frac{1}{4}''$ pitch.

Then $\frac{4}{15}$ is the starting fraction.

Now multiply top and bottom of this fraction by any one number to give the numbers of teeth on wheels which are available.

Thus $$\frac{4 \times 5}{15 \times 5} = \frac{20}{75}.$$

If wheels with 20 teeth and 75 teeth are available, then these wheels with an idle wheel between will satisfy.

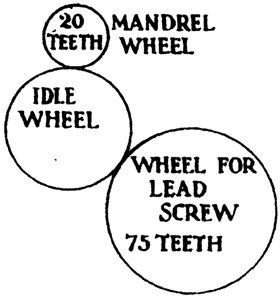


FIG. 120.

The wheel or wheels on top of the fraction are drivers, and those on the bottom are driven wheels.

Fig. 120 shows the set-out for this job.

An idle wheel is used in order that the **lead screw** may revolve in the same direction as the **spindle** or **mandrel**.

Example. *Suppose that one had to cut a screw 15 threads to the inch with a lathe which had a lead screw ½" pitch, and the range of tooth wheels is 20 to 120 in lifts of 5 teeth.*

The beginning fraction is now $\frac{2}{15}$.

$$\frac{2 \times 10}{15 \times 10} = \frac{20}{150}.$$

We have a 20-tooth wheel but not a 150-tooth wheel. Hence we split up the 150 into two parts—one of which will be the number of teeth on one of the wheels. Thus—

$$\frac{20}{50 \times 3}$$

Multiply top and bottom of the latter fraction by 25 ; thus-

$$\frac{20 \times 25}{50 \times 75}$$

On the top of the fraction are the numbers of teeth on the two drivers, and on the bottom are numbers of teeth for the driven wheels.

Fig. 121 shows the set-out as a compound train.

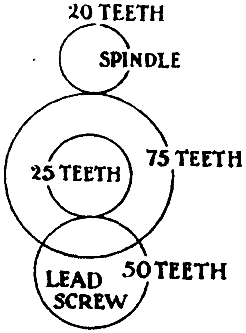


FIG. 121.

Metric Threads

To cut a screw on the centigrade system of measurement, employing a lead screw with a pitch on the inch scale, one needs a special wheel of 127 teeth. This wheel is used as a conversion factor ; but the commencing fraction is the same as in the above examples.

Example. *Find a train of wheels to cut a screw, 7 millimetres pitch, with a lead screw ¼" pitch.*

$$\frac{\text{Threads per inch on lead screw}}{\text{Threads per inch on screw to be cut}} = \frac{4}{\frac{25{\cdot}4}{7}}$$

i.e. there are 25·4 millimetres in one inch.

$$\text{Continuing: } \frac{4 \times 7}{25{\cdot}4} = \frac{4 \times 7}{2 \times 12{\cdot}7} = \frac{4 \times 70}{2 \times 127}$$

$$= \frac{4 \times 35}{127} = \frac{80 \times 35}{127 \times 20}$$

Wheels of 80 teeth and 35 teeth are drivers. Fig. 122 illustrates the set-out.

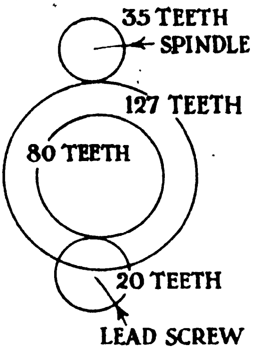


Fig. 122.

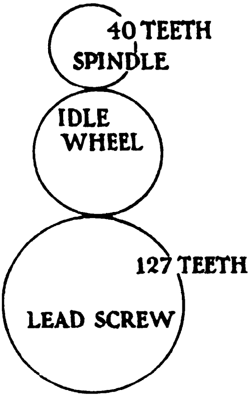


Fig. 123.

Metric Threads

Find a train of wheels to cut a screw, 4 millimetres pitch, with a lathe, the lead screw of which is ½″ pitch.

$$\frac{\text{Threads per inch on lead screw}}{\text{Threads per inch on screw to be cut}} = \frac{2}{\frac{25{\cdot}4}{4}}$$

$$\text{Continuing: } \frac{2 \times 4}{25{\cdot}4} = \frac{4}{12{\cdot}7} = \frac{40}{127}.$$

Fig. 123 illustrates the set-out.

EXERCISE 16

(1) A belt pulley is 21″ diameter and makes 120 revolutions a minute. Calculate the speed of the belt.

✓(2) A line shaft has a belt pulley 18″ diameter and makes 140 revolutions a minute. It drives a pulley on the machine 6″ diameter. How many revolutions per minute will the machine pulley make ?

(3) A worm shaft drives a tooth wheel. The worm shaft makes 1800 revolutions a minute. How many revolutions per minute does the tooth wheel make if it has the following numbers of teeth : (*a*) 90, (*b*) 50, (*c*) 45, (*d*) 120, (*e*) 60 ?

(4) A cone pulley on a line shaft has four steps and makes 120 revolutions per minute. The steps are 6″, 9″, 12″, and 15″ diameter respectively, and the cone pulley on the machine spindle is the same size. Calculate the range of speed in revolutions per minute the machine spindle may make.

(5) A belt has a speed of 1100 ft. per minute. A pulley is to be driven by the belt and to make 140 revolutions per minute. Calculate the diameter of the pulley to be used.

(6) A rope pulley is 36″ effective diameter and makes 400 revolutions a minute. It drives a dynamo armature which must revolve at 1800 revolutions a minute. What must be the effective diameter of the pulley on the armature shaft ?

(7) Two tooth wheels are in gear. The driver has 45 teeth and makes 140 revolutions a minute. The driven wheel has 105 teeth. How many revolutions a minute does it make ?

(8) A driven wheel has to make 55 revolutions a minute and the driver has 65 teeth and makes 110 revolutions a minute. How many teeth must the driven wheel have ?

(9) The two pairs of wheels in the back gear of a lathe have 20 and 80 teeth. What is the reduction ratio of the back gear alone ?

(10) Assuming the back gear as in question (9), what is the cutting speed in feet per minute of a piece of work 14″ diameter ? The lathe cone pulley makes 48 revolutions per minute and the back gear is " in."

(11) The circles A, B, and C represent the wheels at the headstock of a lathe (Fig. 124). A is on the spindle, B is idle, and C is on the lead screw. If A has 25 teeth and C has 105 teeth, how many revolutions will C make while A makes 315 ?

(12) If the lead screw is ¼″ pitch, what is the traverse during the period stated in question (11) ?

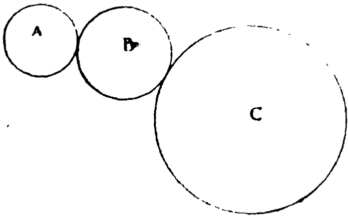


FIG. 124.

(13) If A (Fig. 124) has 35 teeth and makes 140 revolutions a minute, how many teeth should C have so as to make 50 revolutions a minute ?

(14) If C (Fig. 124) should make 60 revolutions a minute and have 120 teeth, how many teeth should A have if it makes 200 revolutions a minute ?

(15) The effective diameter of a milling cutter is 3½″ and makes 45 revolutions a minute. Calculate the cutting speed in feet per minute.

(16) The wheels M, N, P, Q (Fig. 125) represent a compound train at the lathe headstock. M is on the spindle and Q is on the lead screw. N and P are keyed on the same sleeve and revolve together. If M has 20 teeth, N 50, P 25, and Q 120, find the revolutions per minute of Q when M makes 160.

(17) If M (Fig. 125) has 30, N 90, P 35, and Q 105 teeth, what revolutions per minute should M make if it be required that Q make 12 revolutions ?

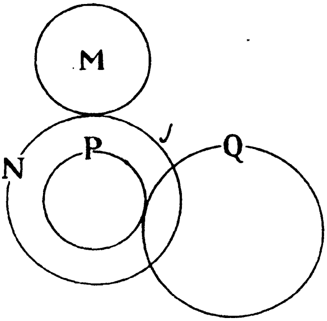


FIG. 125.

(18) What train of wheels is required to cut a screw 8 threads to the inch when the lathe has a lead screw ½″ pitch ? Assume that the lathe has a nest of wheels with teeth 20 to 120 in lifts of 5.

(19) With a lead screw ½″ pitch, what train of wheels would be required to cut a screw 16 threads to the inch ?

(20) What train of wheels is required to cut a screw 4 millimetres pitch ? Lead screw ¼″ pitch. A 127-tooth wheel is allowed.

(21) A pulley revolves at 400 revolutions per minute. What is the angular velocity in radians per second ?

(22) The angular velocity of a shaft is 83·79 radians per second. What revolutions a minute does the shaft make ?

(23) The external radius of a flywheel rim is 10′. When the wheel makes 150 revolutions a minute, what is the linear speed of a point on the rim ?

(24) What is the angular velocity of a wheel revolving at 90 revolutions a minute ?

(25) A piece of work 17½″ diameter is being finished in a lathe, and the revolutions per minute past the tool are 18. Calculate the finishing speed.

(26) The cone pulley of a lathe makes 240 revolutions a minute. The back gear, which has two pairs of wheels 80 and 20 teeth, is in. How many revolutions per minute does the work make ?

(27) If the work in the last example is 3½″ diameter, what is the cutting speed ?

CHAPTER XVII

FRICTION

WHEN one object moves over a flat horizontal surface, an effort E must be applied to keep it in motion at the same speed. This effort is merely overcoming a resistance which is due entirely to **friction.** It will generally be found that a bigger effort is needed to start the motion than is required to keep the object at the same steady speed; that is, the **static friction** is greater than the **kinetic friction.** Wherever there is motion in any object a force must be applied to overcome the resistance due to friction. **Friction is always a retarding force** and, if an external force is not applied to overcome its resisting effect, it will bring the object to rest.

In Fig. 127 the object is shown on the horizontal plane, and the effort is horizontal; in this case the only resistance is the friction between the object and the table. There is friction, however, in every motion, but in most cases there are other resistances besides friction.

Friction is a disadvantage to the engineer in places where lubrication is applied, yet in most cases friction is an advantage, for it is by the aid of friction that the operations are performed.

Advantages due to Friction.	*Disadvantages due to Friction.*
Walking. Clamps. Friction between the driving wheels of locomotives, bicycles, motor-cars, etc., and the road. Holds wedges, cotters, nails, nuts on bolts, in position. Driving machines by belts and ropes. Friction clutches. All brakes.	Any operation requiring lubrication, such as shaft bearings, machine slides, engine sliding parts, etc. The flow of liquids and gases through tubes. Valve rubbers and leathers in pumps.

Coefficient of Friction

In the operation illustrated in Fig. 126 the effort required to start the motion divided by the load is called the **static coefficient of friction,** while the effort required to keep in steady motion divided by the load is called the **kinetic coefficient of friction.** The following table shows the results of experiments carried out with a metal slider on a wood table.

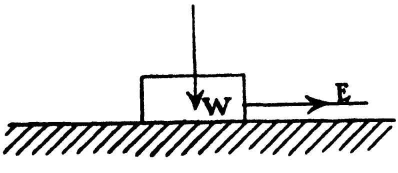


FIG. 126.

Referring to Fig. 126, experiments were carried out on an object weight W lb. The effort x lb. to start the motion, and the effort y lb. to keep in steady motion, are registered.

W lb.	*x lb.*	*Static Coefficient.*	*y lb.*	*Kinetic Coefficient.*
4	3·28	·82	1·8	·45
6	4·86	·81	2·586	·431
8	6·32	·79	3·36	·42
10	8·05	·805	4·41	·441
12	9·6	·8	4·86	·405

It will be seen from the table that the coefficients are fairly constant for various loads. This is the usual principle.

Example (1). *An object of weight* 3 *cwt. rests on a horizontal bench. If the coefficient of friction between the object and the bench is* 0·4, *find the least effort to drag the object along the bench.*

Let E be the effort in pounds. Then—

$$\frac{E}{336} = {\cdot}4$$

$$E = \mathbf{134{\cdot}4\ lb.}$$

Example (2). *An object is held between the jaws of a vice. The vice jaws exert a side pressure of* $\frac{1}{2}$ *ton on the object. If the coefficient of friction between the jaws and the object is* 0·6, *calculate the effort required to move the object.*

Let E be the effort in pounds. Then—

$$\frac{E}{1120} = {\cdot}6$$

$$E = {\cdot}6 \times 1120$$

$$E = \mathbf{672\ lb.}$$

Example (3). *Fig.* 127 *shows a nail in timber. The pressure of the wood on the nail is* 1500 *lb., and the coefficient of static friction for the metal of the nail and timber is* 0·8. *Calculate the frictional holding force of the wood on the nail.*

Let E lb. be the frictional holding force.

$$\frac{E}{1500} = 0{\cdot}8$$

$$E = 0{\cdot}8 \times 1500$$

$$E = \mathbf{1200\ lb.}$$

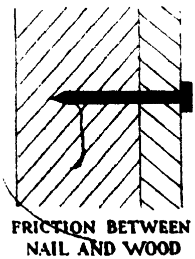


FIG. 127.

The student will observe that when cases arise where the frictional holding force of a nail is not sufficient, then screws or nuts and bolts may be applied.

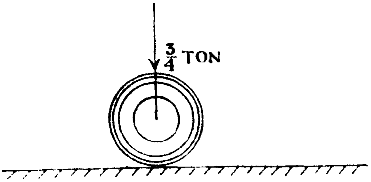


FIG. 128.

Example (4). *In Fig. 128, a motor conveyance driving wheel on the road is shown. If the vertical load on the wheel is $\frac{3}{4}$ ton and the coefficient of friction for the rubber and road is ·75, find the frictional driving force given by the wheel.*

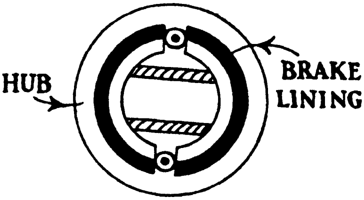
Let E lb. be the driving force. Then—

$$\frac{E}{1680} = \cdot 75$$

$$\therefore E = \cdot 75 \times 1680$$

$$E = \mathbf{1260\ lb.}$$

The student should notice that it is this frictional driving force which urges the conveyance along the road; the same remark applies to all propelled vehicles on roads, such as bicycles, locomotives, traction-engines, etc.



FRICTION ON BRAKE LINING

FIG. 129.

Example (5). *In the brake shown in Fig. 129, let the pressure on the wheel be 1600 lb. and the kinetic coefficient of friction between the brake and wheel lining be* 0·45, *and calculate the braking force.*

Let E lb. be the braking force. Then—

$$\frac{E}{1600} = 0{\cdot}45$$

$$E = 0{\cdot}45 \times 1600$$

$$E = \mathbf{720\ lb.}$$

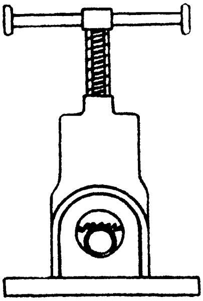


FIG. 130.

Example (6). *Fig. 130 shows a vice holding a tube between its jaws. If the vertical pressure on the tube is 1200 lb., find the frictional holding force on the tube surface.* (Static coefficient of friction is 0·6.)

Let E lb. be the frictional holding force. Then—

$$\frac{E}{1200} = 0{\cdot}6.$$

From which $E = 0{\cdot}6 \times 1200,$

$$E = \mathbf{720\ lb.}$$

Example (7). The frictional force between the driving pulley rim and the belt (Fig. 131) produces the effort by means of which the driven pulley and machines are given their motion.

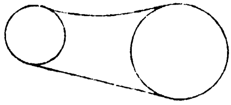

FRICTION OF BELTS AND ROPES IN DRIVES

FIG. 131.

Lubrication

In order to reduce friction in bearings or other sliding parts some form of lubrication is adopted—either by hand or by some automatic process.

Ball bearings or roller bearings are commonly used in order to reduce friction. Ball bearings only need a slight amount of oil and they are very efficient in practice. Care should be taken that the pressure on the steel balls is not too great or the balls will dinge or crack. For heavy loads a suitable bearing must be used.

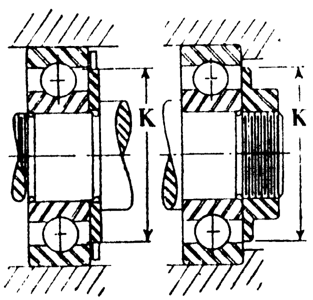


FIG. 132.

Fig. 132 shows a form of ball bearing. The ball race has a diameter equal to K. With this type of bearing only a small amount of lubrication is necessary. Care must be taken, however, to prevent the pressure on the balls being too great, as besides dinging or cracking the balls, the ball races may be deformed. A cracked or broken ball scratches or scores the races and destroys them very quickly.

EXERCISE 17

(1) Give six operations in engineering where friction is a great advantage.

(2) In what ways does an engineer attempt to reduce friction ?

(3) Describe an experiment by which you could obtain the coefficient of friction for two sliding surfaces.

(4) Make sketches of two types of lubricator.

(5) In an experiment between a leather-faced slider on a steel table—polished—the following results were obtained :

Load . . . . . lb.	10	20	30	40
Effort to start motion . . lb.	8·75	17·4	26	35·5
Effort to keep steady motion . lb.	5·1	10·3	15·2	20·2

Find in each case: (*a*) the static coefficient of friction, and (*b*) the kinetic coefficient.

(6) The weight of a motor traction vehicle on its driving wheels is 2 tons and the coefficient of friction between the driving wheels and a flat road is 0·45. What is the greatest effort the tractor can give?

(7) The table of a planing machine and its load weighs 4½ tons. If the coefficient of friction for the sliding table is 0·015, find the effort needed to urge the table along merely against friction.

(8) The side pressure exerted on the end of a hammer shaft between the metal of the hammer and the wood of the shaft is 600 lb. If the coefficient of friction between the metal and wood is 0·45, calculate the effort required to force the hammer head off the shaft.

(9) In the process of keying a pulley to a shaft, the pressure exerted by a jib key on the boss of the pulley is 3½ tons. If the static coefficient of friction between the surface of the key and the pulley boss is 0·35, find the frictional resistance between the key and the boss of the pulley.

(10) In the case of a nut-and-bolt fastening, the pressure between the nut and the washer, when screwed up tight, is 2½ tons. If the static coefficient between the nut and the washer is 0·55, calculate the frictional resistance to the nut becoming loose.

(11) A piece of flat metal is held between the jaws of a vice. The pressure between the vice jaws and the work is 1¾ tons. The static coefficient of friction for the job is 0·062. Find the force needed to move the job while fastened up.

(12) (*a*) A nut is screwed tightly up with a spanner in order to fasten two machine pieces together. (i) What occurs to the shank of the bolt? (ii) What prevents the nut from unscrewing?

(*b*) A small planing-machine table and its load weighs 5 cwt. The coefficient of friction between the table and its slides is 0·02. Calculate the force required to overcome friction.

(13) (*a*) What is meant by "friction"?

(*b*) During an experiment on friction, weights W were pulled along a horizontal table by horizontal efforts E, and these are stated in the table:

W lb.	2	4	6	8	10	12
E lb.	·5	1·1	1·49	2·06	2·52	3·1

Plot a graph of the given figures. From the graph, find the average coefficient of friction for the materials used in the experiment.

(14) A nut and bolt are used as a machine fastening. When screwed up the stretch in the bolt causes a pull of 1 ton to be exerted between the nut and the washer. Take the coefficient of friction to be 0·4 and

calculate the effort required at the end of a spanner of leverage 20″ to commence unscrewing motion. (Average radius of friction circle is $1\frac{1}{8}$ inch.)

(15) A leather-faced slider of weight 20 lb. was on a polished-steel horizontal table. An effort of 15 lb. was needed to start motion to the slider, while 12 lb. kept it in steady motion. State the static coefficient, and the kinetic coefficients of friction.

(16) A load of 900 lb. rests on a horizontal slide. If the coefficient of sliding friction between the load and the slide is 0·266, calculate the minimum force required to move the load.

(17) The tailstock of a lathe weighs 140 lb. If the coefficient of sliding friction for the tailstock is 0·07, find the horizontal force required to move it.

CHAPTER XVIII

MECHANICAL WORK, ENERGY, AND POWER

Mechanical work is done when an effort or a force is exerted through a distance. This effort or force might be exerted in various ways: (1) the cutting force at the tool points of machine tools, (2) by the force of steam, or other gases, on the pistons of engines, (3) by mechanical haulage of vehicles, and so on. Sometimes it may be that the effort moves over a distance, and in other cases the effort or force may be at rest while the surface moves past it; but in any case where there is relative motion mechanical work is being done.

Illustrations of Mechanical Work being done

Case (1). Lathe tool in cutting process. The tool is at rest while the surface of the work passes the point (Fig. 133).

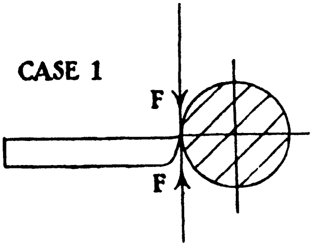


FIG. 133.

Case (2). Tractive forces (Fig. 134). The body shown in Fig. 134 is moved by the force F and in the direction of the force.

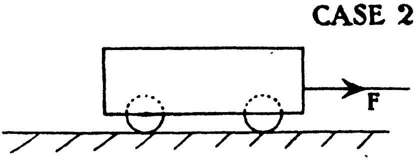


FIG. 134.

Units of Work

The units of work adopted are—

(1) *foot/pound, i.e. ft./lb.*
(2) *foot/tons, i.e. ft./tns.*
(3) *inch/tons, i.e. in./tns.*
(4) *inch/pound, i.e. in./lb.*

Any of these units might be used and they are easily converted from one unit to another. A ft./lb. of work is done when an effort of 1 lb. is exerted through a distance of 1 ft. One ft./lb. of work may be done when an effort of ½ lb. is moved through a distance of 2 ft., or even when an effort of 2 lb. is moved through a distance of ½ ft., so long as the effort multiplied by the distance is equal to one, then 1 ft./lb. of work is done. In general the effort in pounds multiplied by the distance in feet is the number of ft./lb. of work which is done.

Example (1). *A tractive force* (Fig. 134) *of* 150 *lb. is applied to a loaded truck in a works yard and the truck is moved through a distance of* 65 *ft. Calculate the work done.*

Amount of work done = 65 × 150 = **9750 ft./lb.**

Example (2). *The cutting force* (Fig. 133) *of a lathe tool is* 1200 *lb. and the cutting speed is* 90 *ft. per minute. Calculate the amount of mechanical work done in the minute.*

Amount of work done = 90 × 1200 = **108,000 ft./lb.**

Example (3). *A milling tool exerts a cutting force of* 240 *lb. The cutter is* 3½ *in. effective diameter and makes* 60 *revolutions a minute. Calculate the amount of work done in the minute.*

Distance the edge of the cutter moves in a minute

$$= \frac{\overset{11}{\cancel{22}}}{\cancel{7}} \times \frac{\cancel{7} \times \overset{5}{\cancel{60}}}{\cancel{2} \times \cancel{12}} = 55 \text{ ft.}$$

Amount of work done = 240 × 55 = **13,200 ft./lb.**

LIFTING OBJECTS

In lifting objects vertically up, an effort at least equal to the weight of the object must be applied.

Example (4). *A crane lifts a load of* 5 *tons through a distance of* 18 *ft. Calculate the amount of work done—*(*a*) *in ft./tons*, (*b*) *in ft./lb.*

Amount of work done = 5 × 18 = **90 ft./tons.**

or ,, ,, ,, ,, = 5 × 2240 × 18 = **201,600 ft./lb.**

Example (5). *A force of* 900 *lb. is exerted through a distance of* 200 *yds. Find the amount of work done—*(*a*) *in ft./lb.*, (*b*) *in ft./tons.*

Amount of work done = 900 × 600 = **540,000 ft./lb.**

,, ,, ,, ,, $= \dfrac{\overset{27000}{\cancel{540000}}}{\underset{112}{\cancel{2240}}} =$ **242 ft./tons.**

= **242 ft./tons** to the nearest unit.

Power

Power is the Rate at which work is being done—that is, the number of foot/pounds of work being done per minute or per second is a measure of the power being exerted.

The unit of power is the Horse-power, and one horse-power is being accomplished when 33,000 ft./lb. of work per minute is being done.

Example (1). *The cutting force at a lathe is 1400 lb. and the work is 8 in. diameter. If the number of revolutions of the work per minute is 55, calculate the horse-power being absorbed at the tool point.*

$$\text{Cutting speed in ft. per min.} = \frac{22}{7} \times \frac{8}{12} \times 55.$$

$$\text{Work done per minute} = 1400 \times \frac{22}{7} \times \frac{8}{12} \times 55 \text{ ft./lb.}$$

$$\text{Horse-power absorbed} = \frac{\overset{\cancel{200}}{\cancel{1400}}}{1} \times \frac{22}{\underset{1}{\cancel{7}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{12}}} \times \frac{\overset{\cancel{5}}{\cancel{55}}}{\underset{\underset{\underset{3}{\cancel{600}}}{\cancel{3000}}}{\cancel{33000}}}$$

$$= \frac{44}{9} = 4\frac{8}{9}.$$

Example (2). *A planing-machine tool gives a cutting force of 2500 lb., and the cutting speed is 80 ft. per minute. Calculate the horse-power absorbed cutting.*

$$\text{Work done per minute} = 2500 \times 80.$$

$$\text{Horse-power absorbed} = \frac{25\cancel{00} \times 8\cancel{0}}{33\cancel{000}} = \frac{200}{33} = 6{\cdot}06.$$

Example (3). *The tractive force to move a motor-car on a level track at 40 miles per hour is 80 lb. Calculate the horse-power absorbed at this speed.*

$$\text{Distance the car moves in a minute} = \frac{\overset{1760}{\cancel{5280}} \times \overset{2}{\cancel{40}}}{\underset{\cancel{3}}{\cancel{60}}} = 3520 \text{ ft.}$$

$$\text{Work done per minute} = 80 \times 3520 \text{ ft./lb.}$$

$$\text{Horse-power absorbed} = \frac{8\cancel{0} \times 352\cancel{0}}{330\cancel{00}} = \frac{2816}{330}$$

$$= 8{\cdot}53.$$

Energy

Energy is mechanical work which is stored in some object. The storage of energy can be accomplished in a variety of ways:

(1) By means of raised weights, such as in the case of drop hammers, or pile-drivers, or even in loaded trucks at the top of an incline, or in deflected springs.

(2) Energy may be stored in electric batteries or it may be stored in compressed air and in many other ways. When the object is at rest the energy stored is said to be **potential energy.** An example is the self-starter of a motor-car, which uses energy stored in the electric battery to turn the cranks of the engine instead of a man cranking them round.

Kinetic Energy

This kind of energy is stored in an object which is in motion and is due entirely to its being in motion. From the following formula we may calculate the amount of kinetic energy held by any object in motion.

$$\textbf{Kinetic energy} = \frac{\mathbf{WV^2}}{\mathbf{2g}}\ \textbf{ft./lb.},$$

where W is the weight in pounds, V is the velocity or speed in feet per second, $g = 32{\cdot}2$.

Example (1). *The head of a hand hammer is* 2 *lb. in weight and has a speed of* 50 *ft. per second, at the instant of striking. Calculate the amount of kinetic energy in the moving hammer head.*

$$\text{Kinetic energy} = \frac{\cancel{2} \times 50 \times 50}{\cancel{2} \times 32{\cdot}2}$$

$$= \textbf{78 ft./lb. to the nearest unit.}$$

Example (2). *A drop hammer weighs* 15 *cwt. and is raised to a height of* 4 *ft. for the blow. Calculate the amount of potential energy in the raised hammer head.*

$$\text{Potential energy} = 15 \times 112 \times 4\ \textbf{ft./lb.}$$

$$= \textbf{6,720 ft./lb.}$$

Example (3). *The head of a steam hammer and other moving parts weigh* 140 *lb., and at the instant of striking have a speed of* 60 *per sec. Find the amount of kinetic energy stored in the moving parts of the hammer head.*

$$\text{Kinetic energy} = \frac{140 \times \overset{30}{\cancel{60}} \times \overset{30}{\cancel{60}}}{\cancel{2} \times \underset{16{\cdot}1}{\cancel{32{\cdot}2}}}$$

$$= \textbf{7826 ft./lb.}$$

Conservation of Energy

This statement means that energy may be expended in one form, but that it is conserved in another form. For instance, mechanical work is done to drive a dynamo and the dynamo stores electrical energy in batteries. In another case potential energy is stored in a drop hammer and this energy is converted to work done on the job and thence into heat. All fuels possess energy in the form of heat, and science tells us that heat is the source of all energy. It is useful to note that wherever mechanical work is done, heat is produced. In fact, the equivalent of one unit of heat in ft./lb. of work has been obtained very definitely and is given as 778 ft./lb.

Mechanical equivalent of heat = 778 ft./lb.

778 ft./lb. is the equivalent of 1 British thermal unit of heat.

British Thermal Unit is usually abbreviated to B.T.U.

Example (1). *A pound of coal when completely burnt will give out* 13,000 *B.T.U. of heat. To how much mechanical work is this equivalent?*

$$\begin{aligned}\text{Mechanical work} &= 13000 \times 778\\ &= \mathbf{10{,}114{,}000\ ft./lb.}\end{aligned}$$

Example (2). *A lb. of petrol contains* 20,000 *B.T.U.s of heat. To how much mechanical work is this equivalent?*

$$\begin{aligned}\text{Mechanical work} &= 20000 \times 778\\ &= \mathbf{15{,}560{,}000\ ft./lb.}\end{aligned}$$

Friction and Mechanical Work

Since friction is always a retarding force, and further, since there is friction wherever there is motion, it follows that where there is motion then work is absorbed due to the friction.

Example (1). *A planing-machine table and its load weigh* 2 *tons, and the coefficient of friction between the table and slides is* 0·02. *Find the effort required to move the table against friction.*

If the table makes 8 *strokes of* 6 *ft. each per minute, how much work per minute is absorbed?*

$$\begin{aligned}\text{Frictional resistance, } i.e.\text{ effort} &= 2 \times 2240 \times \cdot 02\\ &= \mathbf{89{\cdot}6\ lb.}\\ \text{Work absorbed per minute} &= 89{\cdot}6 \times 48\\ &= \mathbf{4301\ ft./lb.}\end{aligned}$$

Example (2). *A line shaft is* 7" *diameter and makes* 120 *revolutions a minute. The load on a shaft bearing is* 800 *lb. and the coefficient of friction between the surface of the shaft journal and the step of the bearing is* 0·025. *Find the work absorbed per minute* *in friction. Find also the number of British thermal units of heat produced per minute.*

Resistance at journal surface $= 800 \times {\cdot}025$

$$= 800 \times \frac{25}{1000} = 20 \text{ lb.}$$

Work absorbed per minute $= 20 \times \frac{22}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{12}} \times \overset{10}{\cancel{120}}$

$= 4400$ ft./lb.

B.T.U.s produced $= \frac{4400}{778} = 5{\cdot}66.$

Example (3). *Fig.* 135 *represents a friction clutch. Let the total frictional force be* 50 *lb. at a mean radius of* 7″. *If the clutch makes* 2000 *revolutions per minute, calculate the horse-power transmitted by the clutch.*

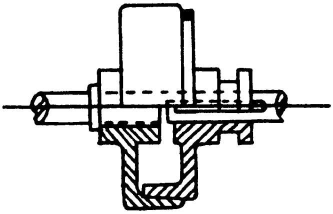


FIG. 135.

Work done per minute $= \frac{22}{7} \times \frac{14}{12} \times 50 \times 2000.$

Horse-power $= \frac{22}{7} \times \frac{14}{12} \times \frac{50 \times 2000}{33000}$

$= 11\frac{1}{9}.$

Diagrams of Work

Often it is very useful to have mechanical work shown in the form of a diagram. Very commonly, when making tests, there is an automatic device for drawing a diagram of work done during the test; especially is this so in obtaining the indicated horse-power of an engine.

Example (1) (Fig. 136). *If a constant effort of* 30 *lb. is exerted through a distance of* 50 *ft., the rectangle shown would represent the diagram of work. The amount of work done is* 1500 *ft./lb., and this number is the area of the diagram according to the scales.*

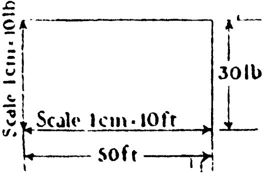


FIG. 136.

In diagrams of work, the effort is usually not constant, and it is this fact which makes the diagram necessary.

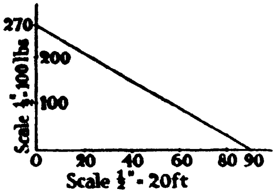


FIG. 137.

Example (2) (Fig. 137). *A chain 90 ft. long, weighing 3 lb. per foot length, hangs vertically, and is then wound completely on a drum. Show the work done in a diagram.*

At the commencement the effort is 270 lb., as all the chain is unwound. When the lower end of the chain is at the heights shown horizontally in the diagram, the height of the graph line represents the effort. The area of the triangle—to scale—represents the work done.

$$I.e.\ \frac{270 \times 90}{2} = 12{,}150 \text{ ft./lb.}$$

Example (3). *Fig. 138 is somewhat like those automatically drawn by the indicator attached to the cylinder of a steam engine.*

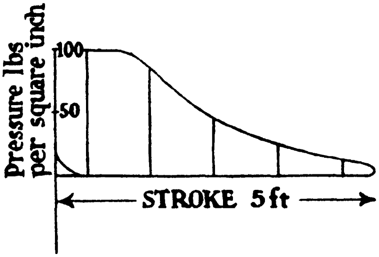


FIG. 138.

The stroke length is shown along the horizontal to scale, *i.e.* equal to 5 ft.

The vertical height, inside of the diagram, represents the pressure per square inch acting on the piston of the engine.

The area of the diagram multiplied by the number of square inches piston area is equal to the work done in the stroke.

To find the area of any figure like the above, use a method named the " mid-ordinate rule."

In this method, one takes a number of equal parts along the base—in the above take 5.

Set up an ordinate at the middle point of each part; *i.e.* a " mid-ordinate." These are shown.

The heights of these to scale are: 100, 83, 40, 25, 17.

To get the average pressure, add the five pressures and divide by 5. Average pressure = 53 lb. per sq. in.

Let the area of the piston = 100 sq. in.

Then 53×100 = average thrust on the piston.

∴ Work done in the stroke = 5300×5 ft./lb.

= 26,500 ft./lb.

Multiply the latter quantity by the number of strokes per minute and this gives the work done per minute by the engine.

Let there be 200 such strokes per minute.

$$\therefore \text{Work done per minute} = 26500 \times 200.$$

$$\text{Horse-power} = \frac{26500 \times 200}{33000} = 160.$$

EXERCISE 18

(1) In mechanics describe what is meant by (*a*) "Work" and (*b*) "Energy." In what units are they measured?

(2) A slotting tool exerts a steady cutting force of 1400 lb. and the length of the stroke is 3 ft. Calculate the amount of work done in the stroke.

(3) A hoist raised a load of 3½ tons from the bottom to the top floor in a works, a distance of 40 ft. Find the amount of work done: (*a*) in ft./tons, and (*b*) in ft./lb.

(4) A drilling machine is turned by hand. The wheel is 21 in. effective diameter and a steady tangential force of 4 lb. is exerted. If a hundred revolutions of the work occur in one minute, how much work has been done?

(5) (*a*) State the "mechanical equivalent of heat."

(*b*) A pound of ordinary coal contains 12,500 British thermal units of heat. To how much mechanical work is this equivalent?

(6) A wire rope at a mine is 400 yards long and weighs 3 lb. per foot length. What work is done in winding up the rope completely?

(7) One cubic foot of coal gas contains 450 B.T.U.s of heat. What does this represent in mechanical work?

(8) What is meant by "the conservation of energy"? Give two examples illustrating your answer.

(9) "Friction causes heat." Explain this statement and how it operates. Give examples to illustrate your answer.

(10) A lathe tool exerts a force of 1100 lb. on a job 21" diameter. The work makes 24 revolutions a minute. (*a*) How much mechanical work is done per minute? (*b*) How many B.T.U.s of heat are produced in the same time?

(11) In question (10), what horse-power is absorbed in the work done?

(12) The moving parts of a steam hammer weigh 8 cwt. and at the instant of striking are moving at 50 ft. a second. Calculate the kinetic energy in the moving parts at this speed.

(13) A truck of weight 10 cwt. is moving at 8 ft. a second. Calculate the amount of kinetic energy in the moving truck.

(14) A drop hammer of weight 15 cwt. is raised to a height of 4 ft. above the job: (*a*) How much potential energy does it hold? (*b*) It is allowed to fall freely, how much kinetic energy is in the hammer head at the instant of striking?

(15) In question (14) the hammer head strikes the surface of a flat piece of metal which sinks 2 in. under the blow. What is the average force of the blow ?

(16) A line-shaft journal is 8″ diameter, and the load on it is ¾ ton. It makes 110 revolutions a minute and the coefficient of friction for the journal and bearing is 0·022. Find the work absorbed per minute in friction.

(17) In question (16) (*a*) what horse-power is absorbed, and (*b*) what heat is produced per minute ?

(18) A crane raises a load of 4 tons to a height of 12 ft. Find the amount of work done : (*a*) in foot/tons, and (*b*) in foot/pounds.

(19) A belt gives an effort of 150 lb. to a driven pulley. The belt speed is 1200 ft. a minute. What amount of mechanical work is being transmitted per minute ?

(20) In question (19), what horse-power is being transmitted ?

(21) The pressure on the teeth of a gear wheel is 120 lb. and the pitch diameter is 17½″. The wheel makes 90 revolutions a minute. What horse-power is being transmitted ?

(22) A planing machine and its load weigh five tons and the coefficient of friction between the table and slides is 0·02. If the table makes 12 strokes of 11 ft. per minute, what amount of work is absorbed per minute in overcoming friction ?

(23) In the planing machine of question (22) there are 6 cutting strokes, and the tool exerts a steady force of 2200 lb. Find the total horse-power used in working the machine.

(24) A planing-machine table and its load weigh 2¾ tons. The coefficient of sliding friction for the table and slides is 0·022. Calculate the effort required to move the table against friction. Also, if the table makes sixteen strokes of 6 ft. per minute, calculate the horse-power absorbed in working the machine against friction.

(25) In a line-shaft bearing, the journal of the shaft is 11″ diameter, and the load on the shaft is 2⅛ tons. The coefficient of friction between the shaft and the steps of the bearing is 0·018. Calculate the surface resistance of the shaft journal. If the shaft makes 150 revolutions a minute, find the foot/pounds of work per minute and the horse-power absorbed by friction in the bearing.

(26) A motor truck in a works yard weighs 24 cwt. and the coefficient of friction between the wheels of the truck and the road is 0·42. Calculate the greatest effort the motor can apply to draw a load of trucks along the road. Also calculate the horse-power exerted by the truck in drawing its maximum load along a level track at 8 miles an hour.

(27) The weight of a locomotive on the driving wheels is 40 tons and the coefficient of friction between a wet rail and the driving wheels is 0·15. Calculate the greatest effort the locomotive can give to draw a load of wagons. Also calculate the horse-power the locomotive is giving when the speed is 15 miles per hour.

(28) The cutting tool of a lathe exerts a cutting force on the work of 800 lb. and the cutting speed is 75 ft. per minute. Calculate the horse-power consumed in cutting. If 2·2 horse-power are supplied to the lathe, how many foot/pounds of work per minute are absorbed in overcoming friction of the lathe ?

(29) In turning a shaft on a lathe, a cut 0·03″ deep and a traverse feed of 0·12″ are being applied when the shaft is 8″ diameter. The cutting force at the tool point is 870 lb. and the revolutions of the work per minute are 35. Calculate the work done per minute and the horse-power absorbed at the tool point. Also calculate the amount of metal removed per horse-power per minute.

(30) A conveyor raises two tons of bolts per hour through a height of 20 ft. Calculate the mechanical work done per minute in foot/pounds.

(31) A brake exerts a frictional force of 30 lb. on a flywheel rim 3½ ft. diameter. What work is absorbed per revolution by the brake ?

(32) In sharpening a chisel on a rotating stone 14″ diameter, the resistance on the rim of the stone is ¼ lb. and the stone is revolving at 1800 per minute. Calculate the work absorbed per minute in sharpening.

CHAPTER XIX

HEAT

Nature of Heat

THE sensations of heat and cold are felt by all and may be detected by the sense of touch or by scientific instruments, for in ordinary circumstances the addition of heat to an object raises its temperature and the loss of heat causes the temperature to fall.

The most common source of heat is the combustion of fuels—that is, a chemical process—but heat may be produced by mechanical means. The cutting tools of lathes and other machines become hot ; the point of a chisel being sharpened becomes hot ; and many similar instances of articles becoming hot when worked upon will occur to the student. Hence there is some relation between mechanical work done and heat produced. Experiment shows that such a relation does exist, and that the relation is a very definite one. **A given quantity of mechanical work done will produce a definite quantity of heat**, however the work may be done on an object. Conversely, from heat, mechanical work can be obtained : a given quantity of heat giving a definite quantity of mechanical work. From these considerations we may judge that **heat is a form of energy, for energy is mechanical work stored up.**

Transfer of Heat

When a pan of cold water is placed on a furnace or fire, the heat from the fire passes into the water. Similarly, when a lathe working tool becomes very hot, suds are poured on to it in order to take away the heat, *i.e.* the heat passes from the tool into the liquid. As heat can pass from one object to another, one is led to infer that heat is a physical quantity which can be measured, for which purpose a " **unit** " **of heat is required.**

Unit of Heat

The British thermal unit of heat *is the amount of heat required to raise one pound of water one degree in temperature on the Fahrenheit scale.* Another unit of heat used largely in scientific experiments is the **calorie** ; this is the amount of heat required to raise one gram of water one degree in temperature on the Centigrade scale.

> **Example.** *Ten British thermal units of heat will raise* 1 *lb. of water* 10° *F., or will raise* 10 *lb. of water* 1° *F., or* 2 *lb. of water* 5° *F., or* 5 *lb. of water* 2° *F.*
>
> I.e., *the weight of water in pounds, multiplied by the rise of temperature Fahrenheit, is equal to the British thermal units of heat used.*

Sensible Heat

The name "sensible heat" is given to heat which raises the temperature of an object. We can gauge the temperature either by the sense of touch or by sensitive scientific instruments, such as the thermometer or pyrometer.

Example. *How many sensible heat units are given to* 10 *lb. of water to raise its temperature from* 57° *F. to* 211° *F.?*

Rise of temperature 154° F.

Heat units given = weight of water × rise of temperature
= 10 × 154 = 1540 B.T.U.

Latent Heat

"**Latent heat" is that heat which is given to an object while the temperature of the object remains constant.** Thus, when water in an open vessel arrives at the boiling temperature, there is no increase of temperature although one continues the supply of heat. Likewise, during the melting operation of metals, the temperature of the metal remains constant during the melting process. Generally, the physical state of the object is changing during the application of latent heat, *i.e.* water is being converted to steam and the metal is changing from solid to liquid.

The latent heat of "fusion" is the number of heat units required to melt unit weight of the material. The latent heat of fusion of some metals is given in the table following:

Name of Metal.	*Latent Heat of Fusion.*	
	B.T.U. per lb.	*Gram Calories, or Calories per Gram.*
Aluminium	152·5	85
Antimony	55·8	31
Bismuth	23·6	13
Chromium	126	70
Cobalt	114·8	64
Copper	76·8	43
Iron	97	54
Lead	10·1	5·6
Magnesium	91·5	51
Manganese	67	37
Monel metal	122·4	68
Silver	39·6	22
Tin	25·2	14
Zinc	47	26

The **latent heat of vaporisation** of water is variable and depends on the temperature of boiling. For the ordinary boiling temperature of water, *i.e.* 212° F., the latent heat required to convert 1 lb. of water into 1 lb. of steam is 966 B.T.U. The latent heat of fusion for 1 lb. of ice to water is 144 B.T.U.

Latent Heat Explained

Fig. 139 illustrates the apparatus for obtaining the latent heat of steam.

APPARATUS TO FIND LATENT HEAT OF STEAM

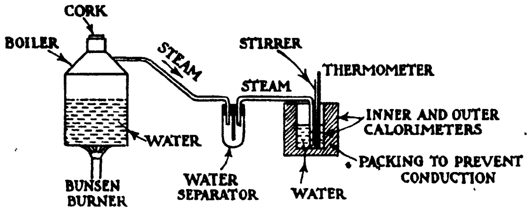


FIG. 139.

In the experiment, cold water is in the calorimeter and steam is passed over to it. The steam is condensed in the cold water, the temperature of which rises.

The following particulars were noted :

Total weight of water, including the water equivalent of the calorimeter	= 250 grams.
Weight of steam passed over	= 8·16 grams.
Temperature of steam passed over	= 99·7° C.
Temperature of cold water, initially	= 4° C.
Final temperature in the calorimeter	= 24° C.

(For note on the water equivalent of a calorimeter, see page 159.)

Let L = latent heat per gram of steam at 99·7° C.

Then 8·16L = total latent heat given up.

Also sensible heat given up by the steam falling in temperature

$$= 8{\cdot}16(99{\cdot}7 - 24) = 8{\cdot}16 \times 75{\cdot}7 = 618.$$

$$\therefore \text{Total heat given up by the steam} = 618 + 8{\cdot}16L.$$

$$\text{Heat received by the cold water} = 250(24 - 4) = 250 \times 20.$$

$$\text{Now, heat given up} = \text{heat received.}$$

$$\therefore 618 + 8{\cdot}16L = 250 \times 20.$$

$$8{\cdot}16L = 250 \times 20 - 618$$

$$8{\cdot}16L = 5000 - 618 = 4382.$$

$$\therefore L = \frac{4302}{8{\cdot}16} = 537 \text{ calories.}$$

The equivalent to this number of calories in B.T.U.s per lb. of water is

$537 \times \frac{9}{5}$, *i.e.* = **966 B.T.U.**

In laboratory work, the gram centimetre system of measurement is more convenient than the lb./ft. system.

The following table gives the latent heat of vaporisation of a few liquids:

Name.	*Calories per gram.*
Methylated spirit	263·8
Ether	90·5
Oil of turpentine	74
Alcohol	202·5
Acetic acid	102
Bromine	45·5
Carbon disulphide	86·7
Ammonia	308–322
Chloroform at 100° C.	81

Multiply each of the numbers in the table by the fraction $\frac{9}{5}$ to get the equivalent British thermal unit number, *i.e.* for the pound/Fahrenheit scales.

SPECIFIC HEAT

The specific heat of a material is the name given to the ratio:

$$\frac{\textbf{Heat required to raise a given weight of material } \mathbf{1^\circ}}{\textbf{Heat required to raise the same weight of water } \mathbf{1^\circ}}.$$

But as the specific heat of water is taken as "one," the easiest way of stating specific heat is:

Specific heat = the amount of heat required to raise one gram, or one pound, of the material one degree in temperature.

The apparatus (Fig. 140) illustrates the method of finding the specific heat of a piece of metal.

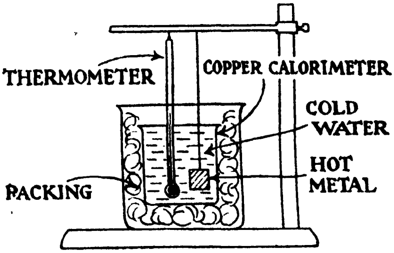


FIG. 140.

Experiment. ***A piece of copper is placed in hot water until it has attained the temperature of the water. It is then removed and placed immediately in the calorimeter. The water is stirred and the highest temperature it achieves is recorded.***

The following particulars were taken :

Weight of copper = 16·65 grams.
Weight of water in calorimeter = 49 grams.
Water equivalent of calorimeter = 2·1 grams.
Initial temperature of copper = 99·5° C.
Initial temperature of water and calorimeter = 12° C.
Final temperature of mixture = 14·5° C.

Find specific heat.

Let specific heat = S.
Heat given up by the metal = heat received by the water.

$$S \times 16{\cdot}65(99{\cdot}5 - 14{\cdot}5) = 51{\cdot}1 \times 2{\cdot}5$$

$$S \times 16{\cdot}65 \times 85 = 51{\cdot}1 \times 2{\cdot}5$$

$$S \times 1415{\cdot}5 = 127{\cdot}75$$

$$S = \frac{127{\cdot}75}{1415{\cdot}5} = {\cdot}0943.$$

The following table shows the specific heats of many other materials. Since the specific heat is merely a ratio as stated above, it is the same number whether on the pound/Fahrenheit or gram/Centigrade scales.

Name of Material.	*Specific Heat.*
Antimony	·051
Brass	·094
Aluminium	·214
Iron (cast)	·114
Copper	·094
Lead	·031
Platinum	·032
Bismuth	·031
Tin	·056
Zinc	·096
Magnesium	·25
Nickel	·109
Silver	·056
Steel	·112
Sulphuric acid	·34
Oil of turpentine	·43
Acetic acid	·66
Ether	·53
Water	1·00

Example. *Find the number of units of heat required to raise 8 lb. of nickel from 55° F. up to the melting temperature 2645° F.*

Units of heat required = specific heat × rise in temperature × weight
= ·109 × 2590 × 8
= 2258 B.T.U

Additional heat required to melt the 8 lb. of nickel

$$= 8 \times \text{latent heat} = 8 \times 114$$
$$= 912 \text{ B.T.U.}$$

Hence the total amount of heat required to raise the temperature of the 8 lb. of nickel and melt it is

$$2258 + 912 = 3170 \text{ B.T.U.}$$

Efficiency of a Furnace

Ten pounds of aluminium were raised from an initial temperature of 60° F. up to melting-point and were completely fused. The weight of oil used was 6 lb. and the heat value of the oil per pound was 18,500 B.T.U. Find the efficiency of the furnace per cent.

The percentage efficiency

$$= \frac{\text{heat required to melt}}{\text{heat used in the furnace}} \times 100$$
$$= 100\left(\frac{\text{specific heat} \times (1216 - 60) \times 10 + \text{latent heat} \times 10}{6 \times 18500}\right)$$
$$= 100\left(\frac{\cdot 214 \times 1156 \times 10 + 152 \cdot 5 \times 10}{6 \times 18500}\right)$$
$$= 3 \cdot 6 \text{ per cent.}$$

Water Equivalent

When any liquid is heated in a container or calorimeter, the container rises to the same temperature as the liquid. Likewise, if the liquid falls in temperature, so does the container. Hence, it is necessary to take the container into account in all calculations of heat values where a container is in operation. It is usual to obtain the weight of water which is equivalent to the container as regards reception or giving up of heat. This is what is meant by the "water equivalent" of an object. In all cases, it may be obtained from the formula :

Water equivalent = specific heat × weight of the material.

Example. *A copper calorimeter weighs* 200 *grams and the specific heat of copper is* 0·094. *Calculate the water equivalent of the calorimeter.*

$$\text{Water equivalent} = 200 \times \cdot 094$$
$$= 18 \cdot 8 \text{ grams.}$$

Calorific or Heat Value of Various Fuels

"Oil fuel," "fuel oil," or **"liquid fuel"** are terms which are commonly used for the same material.

The term **"crude oil"** should not be used for "fuel oil," as "crude oil" still contains the fractions "petrol" and "kerosene."

From " crude oil " the fractions " petrol " and " kerosene or paraffin " are distilled off and " fuel oil " remains.

Name of Fuel.	*Heat Value per lb. B.T.U.*
South Wales anthracite	15,000
Best coal	14,500
Average coal	12,500
Russian fuel oil	19,400
Mexican fuel oil	18,900
Shale fuel oil	18,200
Roumanian residuum	18,900
Petrol	20,000
Spirit methyl	11,000
Coal gas	450 per cu. ft.
Suction producer gas	140 per cu. ft.

The above figures relate to the fuels when they are efficiently burned.

Flash Point

The flash point of an oil is the temperature at which the oil gives off inflammable vapour. It varies from 80° F. for oils used on land to 175° F. for oils used in the British Navy.

Mechanical Equivalent of Heat

Earlier in the chapter we have discussed that heat and work are interchangeable, and it has been found by various experimenters, especially " Joule," that 1 British thermal unit of heat is equivalent to 778 ft./lb. of work.

Example (1). *One pound of a particular coal has a calorific value of* 12,000 *B.T.U. To how much mechanical work is this equivalent?*

$$\text{The work equivalent} = 778 \times 12000$$
$$= \mathbf{9{,}336{,}000\ ft./lb.}$$

Example (2). *A planing-machine tool gives a cutting force of* 2500 *lb. The stroke is* 8 *ft. long. Find the amount of work done per stroke, and also the number of units of heat produced per stroke.*

$$\text{Work done per stroke} = 8 \times 2500 = \mathbf{20{,}000\ ft./lb.}$$

$$\text{Heat units per stroke} = \frac{20000}{778} = \mathbf{25{\cdot}7\ B.T.U.}$$

Example (3). *A line shaft* 7″ *diameter is revolving at* 120 *revolutions a minute, and it transmits a load of* 880 *lb. to a bearing. Find the amount of mechanical work absorbed in friction at the bearing and also the heat produced per minute. (The coefficient of friction at the bearing is* 0·02.)

Mechanical work absorbed per minute

$$= 880 \times \cdot 02 \times \frac{22}{7} \times \frac{7}{12} \times 120$$

$$= 3872 \text{ ft./lb.}$$

$$\text{Heat units produced per minute} = \frac{3872}{778}$$

$$= 5 \text{ approximately.}$$

Transmission of Heat

There are three ways in which heat can be transmitted from one place to another :

(*a*) **Radiation.**
(*b*) **Conduction.**
(*c*) **Convection.**

Radiant heat is transmitted in every direction. If a hot object is suspended from the ceiling of a room, one can feel the heat from it in all positions, *e.g.* either above, below, or on the sides. Science tells us that radiant heat is " vibrations in the ether of space," *e.g.* the radiant heat of the sun gets to us by this process.

Conduction.—Conducted heat is transmitted from particle to particle through the material of the object. In engineering there are good conductors and bad conductors of heat, both of which have their uses. A simple experiment to illustrate conduction is shown in Fig. 141.

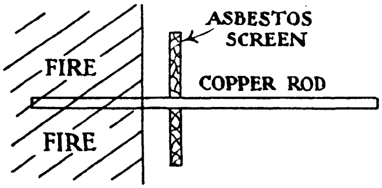


Fig. 141.

In Fig. 141, a copper rod has one end in the fire and a screen of asbestos prevents the heat of the fire from getting to the section of the rod on the right-hand side of the screen. Yet we find the whole length of the rod becomes very hot. This can only occur because the heat is transmitted from particle to particle throughout the length of the rod. If wood had been used instead of the copper rod, we should have found that we could allow it to burn, then push the rod of wood through the sheet until nearly all the wood has burnt, and still be able to hold the wood. From this we find: copper is a good conductor of heat and wood is a very bad conductor.

Good Conductors.		*Bad Conductors.*
Material.	*Relative Conductivity.*	
Silver	100	Wood.
Copper	73·6	Wool.
Gold	53·2	Asbestos.
Aluminium	38·2	Paper.
Brass	23·6	Water.
Zinc	19	Ice.
Tin	14·5	Sand.
Iron	12	Earth.
Steel	11·8	Slag wool.
Lead	8·6	Glass wool.
		Dry gases.

A simple experiment showing relative conductivity of four rods of different materials is shown in Fig. 142.

The lengths of the rods on the left-hand side of the wood screen are placed in a jar of boiling water. Eventually the heat from the water is conducted to the bobs of wax on three of the rods. The bob on the copper rod melts first, that on the aluminium second, and that on the iron (or steel) third. The bob on the asbestos rod does not melt. This simple experiment illustrates that copper is the best conductor, and that asbestos is a very poor conductor, or is an insulator of heat.

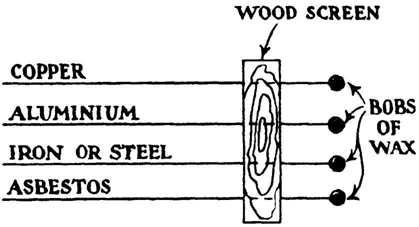


Fig. 142.

Convected Heat.—This type of transmission is obtained by the movement of particles of the material. All fluids, whether liquids or gases, are heated by convection. A simple experiment illustrating convected heat is shown in Fig. 143.

It will be observed, as shown in Fig. 143, that the hot particles at the bottom of the water rise to the top, and the cooler ones at the top sink and take their place. These in turn become hot and rise up in the current. The heated particles rise to the cooler ones above and give some of their heat to the cooler particles. The rising currents of hot water and descending cold currents are called convection currents.

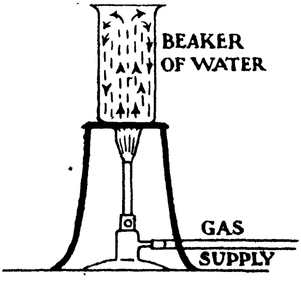


Fig. 143.

The associations in engineering are very numerous:

(1) In the case of central heating by circulating hot water, the water is heated in the basement and convection currents of hot water rise from the basement and circulate through the building, giving heat to the rooms and then returning to the boiler to be heated again.

(2) Convection currents of air (hot) are set up in the shaft of a chimney, and these hot-air currents are what cause the draught to blow up the furnace fire and remove the smoke and furnace gases.

(3) In rare cases, convection currents are used to ventilate mines: furnace gases and flame are allowed to emerge near the top of the upcast shaft of a mine and heat the air near the top of the shaft. This hot air rises and draws the foul air from the mine and fresh air down the downcast shaft, in order that the fresh air shall replace the foul air taken from the mine.

In the case of "conducted heat," besides the amount conducted having some relation to the material, it depends on the difference of temperature between the two sides of the surface, on the area of the surface, and on the thickness through which the heat is coming. The reason for casting fins on a motor-cycle cylinder is for the purpose of creating a large area from which the heat can be taken away. Similarly, a motor-car radiator is made such that a large area of thin material is obtained to get the heat from the hot water within and radiate it to the cooler atmosphere.

Reflection of Heat

The general laws of the "reflection of heat" are the same as those for the reflection of light: brightly polished metals, such as silver, brass, and steel, reflect most of the heat which is directly incident upon them. The reflecting power of any surface increases with the degree of polish of that surface.

RELATIVE REFLECTING POWER OF DIFFERENT SURFACES

Nature of Surface.	*Relative Reflecting Power.*
Polished silver	100
Polished brass	98
Polished speculum metal	88
Polished tin	87
Polished steel	85
White limewash	53

Heat may be brought to a focus just as light can be brought to a focus. In Fig. 144, H represents a source of heat at the focus of the reflector R. Most of this heat is transmitted by reflection to the reflector S, and the

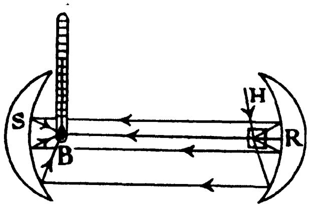


FIG. 144.

heat striking this reflector is brought to a focus at B, where there is the bulb of the thermometer T. The rise of temperature in the thermometer bulb is indicated by a thread of mercury rising up the tube. The concave reflectors shown in Fig. 144 may be of the spherical shape, but would be better if of the paraboloid shape.

A simple experiment showing a convex lens bringing the rays of the sun to a focus at a point O is illustrated in Fig. 145.

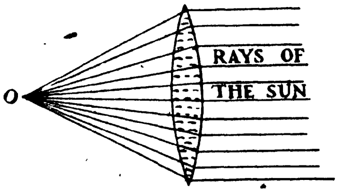


FIG. 145.

In this case the heat from the sun which strikes the lens is only partially reflected and the other part passes through the lens, being brought to a focus. If a piece of paper or thin pine wood is held at the focus, the intensity of the heat there will very often set fire to the substance. The heat passing through the lens is said to be "refracted" by the glass, and this refraction brings the heat to a focus.

Absorption of Heat

Some materials reflect very little heat; any heat incident upon these surfaces is absorbed almost completely into the material.

RELATIVE ABSORBING CAPACITIES

Nature of the Material.	*Relative Absorbing Capacity.*
Lamp black	100
White lime surface . . .	52
Metals (polished) . . .	5 to 14

It is easily observed with black tarmacadam roads that on a hot summer day the surface of the road becomes soft because of the large quantity of heat which has been absorbed by the black surface of the road.

Emission of Heat

Emission of heat by a surface means the heat which is allowed to be radiated from the surface and which comes from the other side of it. Thus, take the radiator of a hot-water circulating system: the hot water is inside the radiator and the heat passes from the hot water through the metal of the radiator and is emitted from the surface into the rooms.

Dull, dark surfaces are the best emitters of heat, while highly polished metal ones are the poorest. The emitting powers of surfaces are almost the same as the absorbing powers.

RELATIVE EMISSIVE POWERS

Nature of the Material.	*Relative Emissive Power.*
Lamp black	100
White lime surface . . .	52
Metals (polished) . . .	5 to 14
Gold leaf.	4 to 5
Mercury spread on a surface .	2 to 3

It will follow that in cases where heat is required to flow easily from an object, a dull black surface will operate best. Thus, for motor-cycle cylinders, the outer surfaces should be dull black in order to get the best cooling results.

EXERCISE 19

(1) Explain what is meant by the terms "Sensible Heat" and "Latent Heat."

(2) In the case of a metal, what is happening to the metal (*a*) when latent heat is being supplied, and (*b*) when latent heat is being given up ?

(3) Describe an experiment for obtaining the latent heat of water to steam. Make a sketch of the apparatus needed.

(4) What is meant by "Specific Heat" ? With the aid of a sketch describe an experiment for obtaining the specific heat of a metal.

(5) A car radiator contains 12 lb. of water at 50° F. before starting up. After having run the car a short time the temperature of the water was 190° F. Calculate the number of British thermal units of heat given to the water and not radiated.

(6) The specific heat of copper is 0·094. Find the amount of heat needed to raise 20 lb. of copper up to its melting temperature, which is 1978° F. from 58° F.

(7) If the latent heat per pound of the copper in question (6) is 77·6, how much latent heat is needed to melt it ?

(8) It took 10 lb. of fuel oil of calorific value 18,000 B.T.U. per lb. to perform the operations in questions (6) and (7) above. Find the efficiency of the furnace used.

(9) What is the "mechanical equivalent of heat" ?

(10) A milling cutter is $3\frac{1}{2}''$ effective cutting diameter and makes 50 revolutions a minute. The cutting force is 1200 lb. Find the mechanical work done per minute and the number of B.T.U.s of heat produced in that time.

(11) A planing-machine table and its load weighs $2\frac{1}{2}$ tons. The stroke

is 10 feet and it makes 8 strokes a minute. The coefficient of friction between the table and slides is 0·03. Find the work absorbed in friction per minute, and the heat units produced.

(12) Why are motor-cycle cylinders usually painted black ?

(13) The roofs of some workshops are sometimes whitewashed on the outside. Why is this ?

(14) How is the heat produced at the working point of a lathe cutting-tool carried away ?

(15) Name six good conductors of heat and six very poor conductors.

(16) Describe a method of preventing heat from escaping from a workshop oven.

(17) What is the difference between " radiant heat " and " reflection of heat " ?

(18) Make a table of " good and poor absorbers of heat."

(19) Why are the ends of good-class steam-engine cylinders polished ?

(20) Make a set-out line sketch of a central-heating system by the circulation of hot water.

(21) Why are " fins " cast on motor-cycle engine cylinders ?

(22) A short steel rod gets hot throughout when one end is held in a flame, yet a handful of steel swarf may be held with some of it in the flame. Why ?

(23) In an experiment to obtain the specific heat of a piece of nickel, the following particulars were taken :

Weight of nickel 300 grams.
Water equivalent of calorimeter 28 grams.
Weight of cold water 2000 grams.
Temperature of cold water 18·1° C.
Temperature of the metal before it is dropped into the water 922° C.
Final temperature of the water and the metal 32·4° C.
Calculate the specific heat of the nickel.

(24) In a test to find the specific heat of brass, the following particulars were noted :

Water equivalent of calorimeter 21 grams.
Weight of cold water 297·6 grams.
Temperature of cold water 18° C.
Weight of brass 510 grams.
Temperature of metal before being dropped in the water 99° C.
Final temperature of water and metal 28° C.
From these particulars, calculate the specific heat of brass.

(25) (*a*) The temperature of water entering a condenser is 60° F., and on leaving the water has a temperature of 180° F. If 128,000 lb. of water pass through the condenser per hour, how many British thermal units of heat are taken by the water per hour ?

(*b*) If the condensed steam gives up an average of 1000 B.T.U.s per lb., what weight of steam is condensed per hour ?

CHAPTER XX

TEMPERATURE

If a person were to touch some objects with the bare hand, he might perceive that they were hot, while on touching others he might consider them cold. In general it might be said that **temperature means the hotness or coldness** of an object as compared with some **normal state,** *i.e. as compared* with the ordinary *blood temperature* of a person in good health. Although our sense of human touch enables us to form an idea of the temperature of an object, yet it is not always trustworthy, for much depends on the condition or state of the hand before touching the object; *i.e.* whether it is in the normal state or not. Then, of course, there are the cases where human touch is out of the question, *i.e.* for high and low temperatures. Hence, some form of scientific instrument is used for measuring temperatures; in general, these instruments are called **thermometers** or temperature meters, and for very high or very low temperatures **pyrometers** are used.

Fluids, whether liquid or gas, expand when their temperature is raised and contract again when the temperature is lowered, and it is this scientific principle upon which thermometers operate. The liquid most commonly used is mercury, but for rather low temperatures alcohol may be used.

Fig. 146 illustrates the usual construction of thermometers. There is a bulb of mercury at the bottom which expands or contracts according as the temperature rises or falls, with the consequent rise or fall of the mercury thread in the tube.

Temperature

The freezing and boiling temperatures stated on the thermometers are just the ordinary temperatures which depend on the fact that there is only the atmospheric pressure on the water.

If the pressure on the water is increased, then the boiling temperature rises and the freezing temperature falls.

If the pressure on the water is reduced, then the boiling temperature falls and the freezing temperature rises.

On the two temperature scales noted in Fig. 146, Fahrenheit and Centigrade, the number of degrees between the ordinary freezing and boiling temperatures of water is 180° on the Fahrenheit scale and 100° on the Centigrade scale.

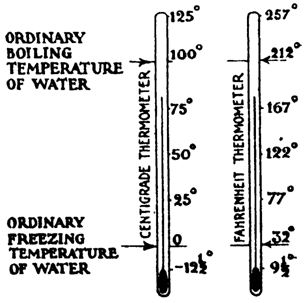


FIG. 146.

Thus 180 degrees F. = 100 degrees C.
or 1 degree F. = $\frac{100}{180}$ or $\frac{5}{9}$ degree C.
Conversely 1 degree C. = $\frac{9}{5}$ degrees F.

To convert from one scale to the other, use is made of these fractions. Thus **to convert from Fahrenheit to Centigrade**:

(Degrees F. — 32) × $\frac{5}{9}$ = degrees C.

Example (1). *Convert* 167° *F. to degrees C.*

$$(167 - 32) \times \tfrac{5}{9} = 75^\circ \text{ C.}$$

Example (2). *Convert* 45° *C. to degrees F.*

$$\text{Degrees C.} \times \tfrac{9}{5} + 32 = \text{degrees F.}$$
$$45 \times \tfrac{9}{5} + 32 = \text{,,}$$
$$81 + 32 = 113^\circ \text{ F.}$$

Example (3). *Mercury freezes at* – 40° *F. Find its freezing temperature in degrees C.*

$$(-40 - 32) \times \tfrac{5}{9} = -40 \text{ C.}$$

Thus the freezing temperature of mercury is at – 40° on both scales.

Example (4). *Liquid air boils at* – 183° *C. Convert this temperature to degrees F.*

$$-183 \times \tfrac{9}{5} + 32 = \text{degrees F.} =$$
$$-297^\circ \text{ F.}$$

Temperature of Furnaces and Hot Metal

The temperature of a hot metal or furnace can often be judged approximately by its colour; the following table gives the approximate temperature for various colours:

	Degrees C.	*Degrees F.*
Dull red	700	1292
Cherry red	900	1652
Orange	1150	2070
White	1300	2372
Dazzling white	1550	2822

The English Steel Corporation give the following instructions for heat-treatment of their tool steels:

TYPES OF STEEL

E. Regular	FORGING	900°–780° C.
	HARDENING	Reheat cutting edge to 760°–780° C. Quench in water.
		Temperature 240°–300° C. light straw colour.

Super A.W. 23	FORGING .	1150°–950° C.
	HARDENING	Reheat cutting edge to 820°–840° C. Air cool or oil quench.
Dux 4 . .	FORGING .	1150°–900° C.
	HARDENING	Heat complete chisel to 870°–900° C. Quench in oil, or to 810°–830° C. Quench in water.

Almost all carbon-steel cutting-tools are " tempered," or, in workshop language, they are " let down," before use. This is because there is probably some brittleness introduced into the tool during the process of manufacture, on account of being quenched suddenly from a high temperature. Consequently, this brittleness is removed by raising the temperature of the tool to a red heat to " soften " the metal. At high temperatures the metal reacts with the oxygen in the air and assumes certain oxides, whose colours correspond to particular temperatures. The operative allows the tool to " air " cool, watching for the particular colour to appear, whereupon he then plunges the tool into the quenching fluid. By plunging the tool when the colour appears, the operative introduces sufficient hardness for the class of work in which the tool has to be used.

Expansion and Contraction due to Temperature Changes

On the bulb of mercury at the bottom of a thermometer (Fig. 146) being raised in temperature, it expands and a thread of mercury is pushed up a very fine bore in the thermometer tube ; the height of the mercury thread registers the temperature. On the bulb cooling, the mercury contracts and the height of the thread in the bore is reduced.

The following are other experiments to show the expansion and contraction due to temperature changes :

The Ball and Ring Experiment (Fig. 147)

The ring was kept cool and, on raising the temperature of the ball, it was found that after a time the ball had expanded and would not pass through the ring although before the ball was heated it would pass through easily. On allowing the ball to cool again, we found it would pass through the ring, showing that the ball had contracted on cooling.

On raising the temperature of the ring and keeping the ball cool, we found that the ball passed much easier through the ring ; showing that the inner diameter of the ring had increased due to the rise in temperature. This scientific principle is used in shrinking steel tyres on locomotive wheels ; in shrinking crank pins into crank arms, etc., etc.

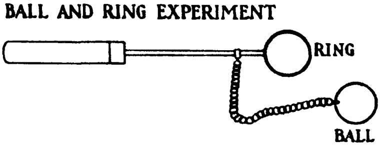


FIG. 147.

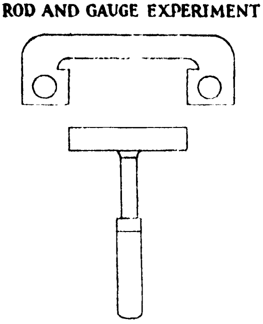


FIG. 148.

Rod and Gauge Experiment (Fig. 148)

When both the rod and gauge were at the same temperature it was found that the rod would just fit the gauge and pass into the hole.

On raising the temperature of the rod and keeping the gauge cool, it was found that the rod would neither fit gauge nor pass into the hole; showing that the expansion had taken place in both directions, *i.e.* length and diameter.

To Show the Expansion of a Liquid (Fig. 149)

On raising the temperature of the water in the flask, the water is seen to rise up the open tube, because of the water expanding.

To Show the Expansion of a Gas (Fig. 150)

On raising the temperature of the air in the flask, the expanding air increases the pressure on the surface of the water and forces some of the water up the open tube.

The temperature of the air can be increased by clasping the upper part of the flask in one's hands.

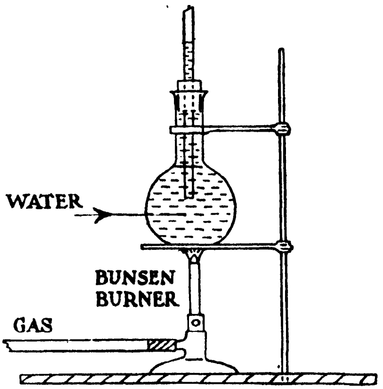


FIG. 149.

The Word " Coefficient "

This word means " multiplying factor "—for instance, in the expression $2{\cdot}5x^2 - 3x$ the coefficient of x^2 is 2·5 and of x is -3. In the following experiment the " coefficient " of linear expansion is found; this coefficient, when multiplied by the length of rod and the rise in temperature of it, will give the total extension of the rod due to the rise in temperature.

Thus if C be the coefficient, L the original length of the rod, T the rise in temperature, and x the total extension of the rod, then

$$x = CLT.$$

Likewise if T were the fall in temperature, then x would be the amount of contraction in length. Thus C is the multiplying factor to the product of L and T.

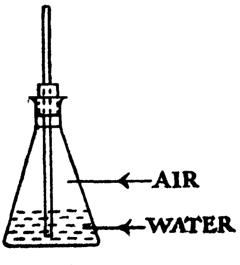


FIG. 150.

Coefficient of Linear Expansion

Linear expansion means *extension in length* (see Fig. 151).

Suppose a rod of metal L inches long is raised in temperature through t degrees. Then the rod will be found to have increased in length x inch.

Then the extension of the rod per degree of temperature $= \frac{x}{t}$.

The extension of 1 in. length per degree of temperature $= \frac{x}{tL}$.

This latter fraction is what is called the **coefficient of linear expansion.**

When we know this for a rod of a particular metal, we can calculate the extension of the rod for any particular rise of temperature and for any length.

The experiment illustrated by Fig. 151 is on a copper tube 40″ long from the clamp to the movable tab B. The tube is clamped at one end but is free to move through the support at the other end. Tab A is fastened to the support and cannot move, while tab B is soldered to the tube and moves with that end. Similar tubes of other metals may be used and the coefficients of expansion found for them.

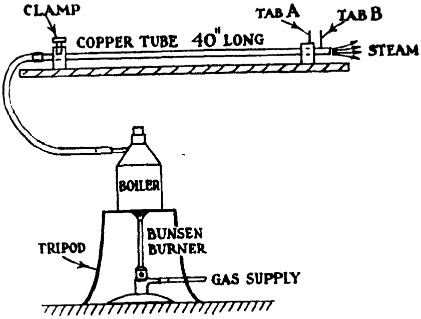


FIG. 151.

Experiment to Find the Coefficient of Thermal Expansion of a Copper Tube (Fig. 151)

In the experiment the water in the boiler is boiled, and steam passes through the tube.

Temperature of tube before passing steam = 61° F.

Distance across tabs A and B before passing steam = ·188″.

Temperature of the issuing steam at the exit = 211° F.

Distance across tabs A and B after passing steam for a time = ·245″.

$$\text{The coefficient of linear expansion} = \frac{\textbf{total extension}}{\textbf{original length} \times \textbf{rise of temp.}}$$

The distance across the tabs A and B was measured with a micrometer, and the total extension was ·245 − ·188 = ·057″.

Original length of tube between tabs A and B = 40″.

Rise of temperature in the tube = 211° − 61° = 150° F.

$$\text{Hence, the coefficient of linear expansion} = \frac{\cdot 057}{40 \times 150} = \cdot 0000095.$$

This is the amount of extension per unit length per degree rise of temperature.

The coefficients of linear expansion for various other metals are included in the following table :

Name of Metal.	*Coefficient of Linear Expansion.*	
	Per degree F.	*Per degree C.*
Aluminium	·0000127	·000023
Brass	·0000104	·0000187
Cast iron	·0000058	·0000106
Copper	·0000095	·0000169
Lead	·0000163	·0000293
Nickel	·0000071	·0000128
Platinum	·0000051	·0000091
Silver	·0000106	·0000191
Steel	·0000067	·0000124
Tin	·0000124	·0000223
Type metal	·0000106	·0000190
White metal—83% tin, 12% antimony, 5% copper	·0000134	·0000242
Zinc	·0000161	·0000292

Example (1). *An aluminium link is* 5 *ft. long and rises in temperature from* 50° *F. to* 130° *F. Calculate its increase in length.*

Let x be the extension :

$$\cdot 0000127 = \frac{x}{60 \times 80}.$$

$$\therefore x = \cdot 0000127 \times 60 \times 80.$$

$$x = \cdot 06096''.$$

Example (2). *A locomotive wheel tyre* 4′ 6″ *internal diameter at* 60° *F. is placed in an oven and raised in temperature to* 310° *F. The coefficient of linear expansion for this particular steel is* 0·0000073. *Calculate the increase in diameter.*

With metal hoops the problem may be solved by taking a length of metal equal to the diameter, for the extension of the circumference is merely π times the extension of the diameter.

Let x be the extension of the diameter :

$$\cdot 0000073 = \frac{x}{250 \times 54}$$

$$x = \cdot 09855''$$

E.g. $\frac{99}{1000}$ in.—that is, over $\frac{3}{32}$ in.

This property is made use of in the operation of " Shrinking On " of locomotive wheel tyres. The wheel tyre is turned so that the internal diameter is slightly less than the diameter of the wheel core. The tyre is then raised in temperature such that the internal

(12) (*a*) Describe an experiment you have performed to show the effect of (i) raising the temperature of liquids, and (ii) cooling them.

(*b*) The internal diameter of a steel hoop is 60″, and its temperature is raised from 15° C. to 125° C. Will the internal diameter increase or decrease? If so, by how much? (The coefficient of linear thermal expansion of steel is ·0000124 per degree C.)

(13) A crank has to be shrunk into its position on the shaft. The hole in it is 12″ diameter at 60° F. The shaft has a diameter of 12·03″ at that temperature. Calculate the temperature to which the crank must be raised so that it may be just pushed on the shaft at 60° F. (Linear coefficient of expansion = ·0000067 per degree F.)

(14) (*a*) Describe an experiment you have performed on the behaviour of metals when they are raised in temperature.

(*b*) State two cases where this property is very useful to the engineer.

(15) (*a*) What is meant by temperature?

(*b*) Sketch the Fahrenheit thermometer, and state how it operates. Mark on it the ordinary freezing and boiling temperatures of water.

(*c*) Copper melts at a temperature of 1985° F. Calculate its melting temperature on the Centigrade scale.

LOGARITHMS.

No.	Log.	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1·0	·0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
1·1	·0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
1·2	·0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
1·3	·1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
1·4	·1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
1·5	·1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
1·6	·2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
1·7	·2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
1·8	·2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
1·9	·2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
2·0	·3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
2·1	·3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
2·2	·3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
2·3	·3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
2·4	·3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
2·5	·3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
2·6	·4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
2·7	·4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
2·8	·4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
2·9	·4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
3·0	·4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
3·1	·4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
3·2	·5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
3·3	·5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
3·4	·5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
3·5	·5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
3·6	·5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
3·7	·5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
3·8	·5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
3·9	·5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
4·0	·6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
4·1	·6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
4·2	·6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
4·3	·6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
4·4	·6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
4·5	·6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
4·6	·6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
4·7	·6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
4·8	·6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
4·9	·6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
5·0	·6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
5·1	·7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
5·2	·7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
5·3	·7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
5·4	·7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

LOGARITHMS.

No.	Log.	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
5·5	·7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
5·6	·7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
5·7	·7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
5·8	·7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
5·9	·7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
6·0	·7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
6·1	·7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
6·2	·7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
6·3	·7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
6·4	·8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
6·5	·8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
6·6	·8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
6·7	·8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
6·8	·8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
6·9	·8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
7·0	·8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
7·1	·8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
7·2	·8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
7·3	·8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
7·4	·8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
7·5	·8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
7·6	·8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
7·7	·8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
7·8	·8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
7·9	·8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
8·0	·9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
8·1	·9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
8·2	·9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
8·3	·9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
8·4	·9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
8·5	·9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
8·6	·9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
8·7	·9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
8·8	·9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
8·9	·9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
9·0	·9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
9·1	·9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
9·2	·9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
9·3	·9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
9·4	·9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
9·5	·9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
9·6	·9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
9·7	·9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
9·8	·9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
9·9	·9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

ANTI-LOGARITHMS.

Log.	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

ANTI-LOGARITHMS.

Log.	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

NATURAL SINES.

Angle	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
0°	·0000	·0017	·0035	·0052	·0070	·0087	·0105	·0122	·0140	·0157	3	6	9	12	15
1°	·0175	·0192	·0209	·0227	·0244	·0262	·0279	·0297	·0314	·0332	3	6	9	12	15
2°	·0349	·0366	·0384	·0401	·0419	·0436	·0454	·0471	·0488	·0506	3	6	9	12	15
3°	·0523	·0541	·0558	·0576	·0593	·0610	·0628	·0645	·0663	·0680	3	6	9	12	15
4°	·0698	·0715	·0732	·0750	·0767	·0785	·0802	·0819	·0837	·0854	3	6	9	12	14
5°	·0872	·0889	·0906	·0924	·0941	·0958	·0976	·0993	·1011	·1028	3	6	9	12	14
6°	·1045	·1063	·1080	·1097	·1115	·1132	·1149	·1167	·1184	·1201	3	6	9	12	14
7°	·1219	·1236	·1253	·1271	·1288	·1305	·1323	·1340	·1357	·1374	3	6	9	12	14
8°	·1392	·1409	·1426	·1444	·1461	·1478	·1495	·1513	·1530	·1547	3	6	9	12	14
9°	·1564	·1582	·1599	·1616	·1633	·1650	·1668	·1685	·1702	·1719	3	6	9	12	14
10°	·1736	·1754	·1771	·1788	·1805	·1822	·1840	·1857	·1874	·1891	3	6	9	11	14
11°	·1908	·1925	·1942	·1959	·1977	·1994	·2011	·2028	·2045	·2062	3	6	9	11	14
12°	·2079	·2096	·2113	·2130	·2147	·2164	·2181	·2198	·2215	·2233	3	6	9	11	14
13°	·2250	·2267	·2284	·2300	·2317	·2334	·2351	·2368	·2385	·2402	3	6	8	11	14
14°	·2419	·2436	·2453	·2470	·2487	·2504	·2521	·2538	·2554	·2571	3	6	8	11	14
15°	·2588	·2605	·2622	·2639	·2656	·2672	·2689	·2706	·2723	·2740	3	6	8	11	14
16°	·2756	·2773	·2790	·2807	·2823	·2840	·2857	·2874	·2890	·2907	3	6	8	11	14
17°	·2924	·2940	·2957	·2974	·2990	·3007	·3024	·3040	·3057	·3074	3	6	8	11	14
18°	·3090	·3107	·3123	·3140	·3156	·3173	·3190	·3206	·3223	·3239	3	6	8	11	14
19°	·3256	·3272	·3289	·3305	·3322	·3338	·3355	·3371	·3387	·3404	3	5	8	11	14
20°	·3420	·3437	·3453	·3469	·3486	·3502	·3518	·3535	·3551	·3567	3	5	8	11	14
21°	·3584	·3600	·3616	·3633	·3649	·3665	·3681	·3697	·3714	·3730	3	5	8	11	14
22°	·3746	·3762	·3778	·3795	·3811	·3827	·3843	·3859	·3875	·3891	3	5	8	11	14
23°	·3907	·3923	·3939	·3955	·3971	·3987	·4003	·4019	·4035	·4051	3	5	8	11	14
24°	·4067	·4083	·4099	·4115	·4131	·4147	·4163	·4179	·4195	·4210	3	5	8	11	13
25°	·4226	·4242	·4258	·4274	·4289	·4305	·4321	·4337	·4352	·4368	3	5	8	11	13
26°	·4384	·4399	·4415	·4431	·4446	·4462	·4478	·4493	·4509	·4524	3	5	8	10	13
27°	·4540	·4555	·4571	·4586	·4602	·4617	·4633	·4648	·4664	·4679	3	5	8	10	13
28°	·4695	·4710	·4726	·4741	·4756	·4772	·4787	·4802	·4818	·4833	3	5	8	10	13
29°	·4848	·4863	·4879	·4894	·4909	·4924	·4939	·4955	·4970	·4985	3	5	8	10	13
30°	·5000	·5015	·5030	·5045	·5060	·5075	·5090	·5105	·5120	·5135	3	5	8	10	13
31°	·5150	·5165	·5180	·5195	·5210	·5225	·5240	·5255	·5270	·5284	2	5	7	10	12
32°	·5299	·5314	·5329	·5344	·5358	·5373	·5388	·5402	·5417	·5432	2	5	7	10	12
33°	·5446	·5461	·5476	·5490	·5505	·5519	·5534	·5548	·5563	·5577	2	5	7	10	12
34°	·5592	·5606	·5621	·5635	·5650	·5664	·5678	·5693	·5707	·5721	2	5	7	10	12
35°	·5736	·5750	·5764	·5779	·5793	·5807	·5821	·5835	·5850	·5864	2	5	7	9	12
36°	·5878	·5892	·5906	·5920	·5934	·5948	·5962	·5976	·5990	·6004	2	5	7	9	12
37°	·6018	·6032	·6046	·6060	·6074	·6088	·6101	·6115	·6129	·6143	2	5	7	9	12
38°	·6157	·6170	·6184	·6198	·6211	·6225	·6239	·6252	·6266	·6280	2	5	7	9	11
39°	·6293	·6307	·6320	·6334	·6347	·6361	·6374	·6388	·6401	·6414	2	4	7	9	11
40°	·6428	·6441	·6455	·6468	·6481	·6494	·6508	·6521	·6534	·6547	2	4	7	9	11
41°	·6561	·6574	·6587	·6600	·6613	·6626	·6639	·6652	·6665	·6678	2	4	7	9	11
42°	·6691	·6704	·6717	·6730	·6743	·6756	·6769	·6782	·6794	·6807	2	4	6	9	11
43°	·6820	·6833	·6845	·6858	·6871	·6884	·6896	·6909	·6921	·6934	2	4	6	8	11
44°	·6947	·6959	·6972	·6984	·6997	·7009	·7022	·7034	·7046	·7059	2	4	6	8	10

NATURAL SINES.

Angle.	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
45°	·7071	·7083	·7096	·7108	·7120	·7133	·7145	·7157	·7169	·7181	2	4	6	8	10
46°	·7193	·7206	·7218	·7230	·7242	·7254	·7266	·7278	·7290	·7302	2	4	6	8	10
47°	·7314	·7325	·7337	·7349	·7361	·7373	·7385	·7396	·7408	·7420	2	4	6	8	10
48°	·7431	·7443	·7455	·7466	·7478	·7490	·7501	·7513	·7524	·7536	2	4	6	8	10
49°	·7547	·7559	·7570	·7581	·7593	·7604	·7615	·7627	·7638	·7649	2	4	6	8	9
50°	·7660	·7672	·7683	·7694	·7705	·7716	·7727	·7738	·7749	·7760	2	4	6	7	9
51°	·7771	·7782	·7793	·7804	·7815	·7826	·7837	·7848	·7859	·7869	2	4	5	7	9
52°	·7880	·7891	·7902	·7912	·7923	·7934	·7944	·7955	·7965	·7976	2	4	5	7	9
53°	·7986	·7997	·8007	·8018	·8028	·8039	·8049	·8059	·8070	·8080	2	3	5	7	9
54°	·8090	·8100	·8111	·8121	·8131	·8141	·8151	·8161	·8171	·8181	2	3	5	7	8
55°	·8192	·8202	·8211	·8221	·8231	·8241	·8251	·8261	·8271	·8281	2	3	5	7	8
56°	·8290	·8300	·8310	·8320	·8329	·8339	·8348	·8358	·8368	·8377	2	3	5	6	8
57°	·8387	·8396	·8406	·8415	·8425	·8434	·8443	·8453	·8462	·8471	2	3	5	6	8
58°	·8480	·8490	·8499	·8508	·8517	·8526	·8536	·8545	·8554	·8563	2	3	5	6	8
59°	·8572	·8581	·8590	·8599	·8607	·8616	·8625	·8634	·8643	·8652	1	3	4	6	7
60°	·8660	·8669	·8678	·8686	·8695	·8704	·8712	·8721	·8729	·8738	1	3	4	6	7
61°	·8746	·8755	·8763	·8771	·8780	·8788	·8796	·8805	·8813	·8821	1	3	4	6	7
62°	·8829	·8838	·8846	·8854	·8862	·8870	·8878	·8886	·8894	·8902	1	3	4	5	7
63°	·8910	·8918	·8926	·8934	·8942	·8949	·8957	·8965	·8973	·8980	1	3	4	5	6
64°	·8988	·8996	·9003	·9011	·9018	·9026	·9033	·9041	·9048	·9056	1	3	4	5	6
65°	·9063	·9070	·9078	·9085	·9092	·9100	·9107	·9114	·9121	·9128	1	2	4	5	6
66°	·9135	·9143	·9150	·9157	·9164	·9171	·9178	·9184	·9191	·9198	1	2	3	5	6
67°	·9205	·9212	·9219	·9225	·9232	·9239	·9245	·9252	·9259	·9265	1	2	3	4	6
68°	·9272	·9278	·9285	·9291	·9298	·9304	·9311	·9317	·9323	·9330	1	2	3	4	5
69°	·9336	·9342	·9348	·9354	·9361	·9367	·9373	·9379	·9385	·9391	1	2	3	4	5
70°	·9397	·9403	·9409	·9415	·9421	·9426	·9432	·9438	·9444	·9449	1	2	3	4	5
71°	·9455	·9461	·9466	·9472	·9478	·9483	·9489	·9494	·9500	·9505	1	2	3	4	5
72°	·9511	·9516	·9521	·9527	·9532	·9537	·9542	·9548	·9553	·9558	1	2	3	3	4
73°	·9563	·9568	·9573	·9578	·9583	·9588	·9593	·9598	·9603	·9608	1	2	2	3	4
74°	·9613	·9617	·9622	·9627	·9632	·9636	·9641	·9646	·9650	·9655	1	2	2	3	4
75°	·9659	·9664	·9668	·9673	·9677	·9681	·9686	·9690	·9694	·9699	1	1	2	3	4
76°	·9703	·9707	·9711	·9715	·9720	·9724	·9728	·9732	·9736	·9740	1	1	2	3	3
77°	·9744	·9748	·9751	·9755	·9759	·9763	·9767	·9770	·9774	·9778	1	1	2	3	3
78°	·9781	·9785	·9789	·9792	·9796	·9799	·9803	·9806	·9810	·9813	1	1	2	2	3
79°	·9816	·9820	·9823	·9826	·9829	·9833	·9836	·9839	·9842	·9845	1	1	2	2	3
80°	·9848	·9851	·9854	·9857	·9860	·9863	·9866	·9869	·9871	·9874	0	1	1	2	2
81°	·9877	·9880	·9882	·9885	·9888	·9890	·9893	·9895	·9898	·9900	0	1	1	2	2
82°	·9903	·9905	·9907	·9910	·9912	·9914	·9917	·9919	·9921	·9923	0	1	1	2	2
83°	·9925	·9928	·9930	·9932	·9934	·9936	·9938	·9940	·9942	·9943	0	1	1	1	2
84°	·9945	·9947	·9949	·9951	·9952	·9954	·9956	·9957	·9959	·9960	0	1	1	1	1
85°	·9962	·9963	·9965	·9966	·9968	·9969	·9971	·9972	·9973	·9974	0	0	1	1	1
86°	·9976	·9977	·9978	·9979	·9980	·9981	·9982	·9983	·9984	·9985	0	0	1	1	1
87°	·9986	·9987	·9988	·9989	·9990	·9990	·9991	·9992	·9993	·9993	0	0	0	1	1
88°	·9994	·9995	·9995	·9996	·9996	·9997	·9997	·9997	·9998	·9998	0	0	0	0	0
89°	·9998	·9999	·9999	·9999	·9999	1·000	1·000	1·000	1·000	1·000	0	0	0	0	0

NATURAL COSINES.

Subtract Differences

Angle	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
0°	1·0000	1·000	1·000	1·000	1·000	1·000	·9999	·9999	·9999	·9999	0	0	0	0	0
1°	·9998	·9998	·9998	·9997	·9997	·9997	·9996	·9996	·9995	·9995	0	0	0	0	0
2°	·9994	·9993	·9993	·9992	·9991	·9990	·9990	·9989	·9988	·9987	0	0	0	0	0
3°	·9986	·9985	·9984	·9983	·9982	·9981	·9980	·9979	·9978	·9977	0	0	1	1	1
4°	·9976	·9974	·9973	·9972	·9971	·9969	·9968	·9966	·9965	·9963	0	0	1	1	1
5°	·9962	·9960	·9959	·9957	·9956	·9954	·9952	·9951	·9949	.9947	0	1	1	1	1
6°	·9945	·9943	·9942	·9940	·9938	·9936	·9934	·9932	·9930	·9928	0	1	1	1	2
7°	·9925	·9923	·9921	·9919	·9917	·9914	·9912	·9910	·9907	·9905	0	1	1	2	2
8°	·9903	·9900	·9898	·9895	·9893	·9890	·9888	·9885	·9882	·9880	0	1	1	2	2
9°	·9877	·9874	·9871	·9869	·9866	·9863	·9860	·9857	·9854	·9851	0	1	1	2	2
10°	·9848	·9845	·9842	·9839	·9836	·9833	·9829	·9826	·9823	·9820	1	1	2	2	3
11°	·9816	·9813	·9810	·9806	·9803	·9799	·9796	·9792	·9789	·9785	1	1	2	2	3
12°	·9781	·9778	·9774	·9770	·9767	·9763	·9759	·9755	·9751	·9748	1	1	2	3	3
13°	·9744	·9740	·9736	·9732	·9728	·9724	·9720	·9715	·9711	·9707	1	1	2	3	3
14°	·9703	·9699	·9694	·9690	·9686	·9681	·9677	·9673	·9668	·9664	1	1	2	3	4
15°	·9659	·9655	·9650	·9646	·9641	·9636	·9632	·9627	·9622	·9617	1	2	2	3	4
16°	·9613	·9608	·9603	·9598	·9593	·9588	·9583	·9578	·9573	·9568	1	2	2	3	4
17°	·9563	·9558	·9553	·9548	·9542	·9537	·9532	·9527	·9521	·9516	1	2	3	4	4
18°	·9511	·9505	·9500	·9494	·9489	·9483	·9478	·9472	·9466	·9461	1	2	3	4	5
19°	·9455	·9449	·9444	·9438	·9432	·9426	·9421	·9415	·9409	·9403	1	2	3	4	5
20°	·9397	·9391	·9385	·9379	·9373	·9367	·9361	·9354	·9348	·9342	1	2	3	4	5
21°	·9336	·9330	·9323	·9317	·9311	·9304	·9298	·9291	·9285	·9278	1	2	3	4	5
22°	·9272	·9265	·9259	·9252	·9245	·9239	·9232	·9225	·9219	·9212	1	2	3	4	6
23°	·9205	·9198	·9191	·9184	·9178	·9171	·9164	·9157	·9150	·9143	1	2	3	5	6
24°	·9135	·9128	·9121	·9114	·9107	·9100	·9092	·9085	·9078	·9070	1	2	4	5	6
25°	·9063	·9056	·9048	·9041	·9033	·9026	·9018	·9011	·9003	·8996	1	3	4	5	6
26°	·8988	·8980	·8973	·8965	·8957	·8949	·8942	·8934	·8926	·8918	1	3	4	5	6
27°	·8910	·8902	·8894	·8886	·8878	·8870	·8862	·8854	·8846	·8838	1	3	4	5	7
28°	·8829	·8821	·8813	·8805	·8796	·8788	·8780	·8771	·8763	·8755	1	3	4	6	7
29°	·8746	·8738	·8729	·8721	·8712	·8704	·8695	·8686	·8678	·8669	1	3	4	6	7
30°	·8660	·8652	·8643	·8634	·8625	·8616	·8607	·8599	·8590	·8581	1	3	4	6	7
31°	·8572	·8563	·8554	·8545	·8536	·8526	·8517	·8508	·8499	·8490	2	3	5	6	8
32°	·8480	·8471	·8462	·8453	·8443	·8434	·8425	·8415	·8406	·8396	2	3	5	6	8
33°	·8387	·8377	·8368	·8358	·8348	·8339	·8329	·8320	·8310	·8300	2	3	5	6	8
34°	·8290	·8281	·8271	·8261	·8251	·8241	·8231	·8221	·8211	·8202	2	3	5	7	8
35°	·8192	·8181	·8171	·8161	·8151	·8141	·8131	·8121	·8111	·8100	2	3	5	7	8
36°	·8090	·8080	·8070	·8059	·8049	·8039	·8028	·8018	·8007	·7997	2	3	5	7	9
37°	·7986	·7976	·7965	·7955	·7944	·7934	·7923	·7912	·7902	·7891	2	4	5	7	9
38°	·7880	·7869	·7859	·7848	·7837	·7826	·7815	·7804	·7793	·7782	2	4	5	7	9
39°	·7771	·7760	·7749	·7738	·7727	·7716	·7705	·7694	·7683	·7672	2	4	6	7	9
40°	·7660	·7649	·7638	·7627	·7615	·7604	·7593	·7581	·7570	·7559	2	4	6	8	9
41°	·7547	·7536	·7524	·7513	·7501	·7490	·7478	·7466	·7455	·7443	2	4	6	8	10
42°	·7431	·7420	·7408	·7396	·7385	·7373	·7361	·7349	·7337	·7325	2	4	6	8	10
43°	·7314	·7302	·7290	·7278	·7266	·7254	·7242	·7230	·7218	·7206	2	4	6	8	10
44°	·7193	·7181	·7169	·7157	·7145	·7133	·7120	·7108	·7096	·7083	2	4	6	8	10

NATURAL COSINES.

Subtract Differences.

Angle	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
45°	·7071	·7059	·7046	·7034	·7022	·7009	·6997	·6984	·6972	·6959	2	4	6	8	10
46°	·6947	·6934	·6921	·6909	·6896	·6884	·6871	·6858	·6845	·6833	2	4	6	8	11
47°	·6820	·6807	·6794	·6782	·6769	·6756	·6743	·6730	·6717	·6704	2	4	6	9	11
48°	·6691	·6678	·6665	·6652	·6639	·6626	·6613	·6600	·6587	·6574	2	4	7	9	11
49°	·6561	·6547	·6534	·6521	·6508	·6494	·6481	·6468	·6455	·6441	2	4	7	9	11
50°	·6428	·6414	·6401	·6388	·6374	·6361	·6347	·6334	·6320	·6307	2	4	7	9	11
51°	·6293	·6280	·6266	·6252	·6239	·6225	·6211	·6198	·6184	·6170	2	5	7	9	11
52°	·6157	·6143	·6129	·6115	·6101	·6088	·6074	·6060	·6046	·6032	2	5	7	9	12
53°	·6018	·6004	·5990	·5976	·5962	·5948	·5934	·5920	·5906	·5892	2	5	7	9	12
54°	·5878	·5864	·5850	·5835	·5821	·5807	·5793	·5779	·5764	·5750	2	5	7	9	12
55°	·5736	·5721	·5707	·5693	·5678	·5664	·5650	·5635	·5621	·5606	2	5	7	10	12
56°	·5592	·5577	·5563	·5548	·5534	·5519	·5505	·5490	·5476	·5461	2	5	7	10	12
57°	·5446	·5432	·5417	·5402	·5388	·5373	·5358	·5344	·5329	·5314	2	5	7	10	12
58°	·5299	·5284	·5270	·5255	·5240	·5225	·5210	·5195	·5180	·5165	2	5	7	10	12
59°	·5150	·5135	·5120	·5105	·5090	·5075	·5060	·5045	·5030	·5015	3	5	8	10	13
60°	·5000	·4985	·4970	·4955	·4939	·4924	·4909	·4894	·4879	·4863	3	5	8	10	13
61°	·4848	·4833	·4818	·4802	·4787	·4772	·4756	·4741	·4726	·4710	3	5	8	10	13
62°	·4695	·4679	·4664	·4648	·4633	·4617	·4602	·4586	·4571	·4555	3	5	8	10	13
63°	·4540	·4524	·4509	·4493	·4478	·4462	·4446	·4431	·4415	·4399	3	5	8	10	13
64°	·4384	·4368	·4352	·4337	·4321	·4305	·4289	·4274	·4258	·4242	3	5	8	11	13
65°	·4226	·4210	·4195	·4179	·4163	·4147	·4131	·4115	·4099	·4083	3	5	8	11	13
66°	·4067	·4051	·4035	·4019	·4003	·3987	·3971	·3955	·3939	·3923	3	5	8	11	13
67°	·3907	·3891	·3875	·3859	·3843	·3827	·3811	·3795	·3778	·3762	3	5	8	11	13
68°	·3746	·3730	·3714	·3697	·3681	·3665	·3649	·3633	·3616	·3600	3	5	8	11	14
69°	·3584	·3567	·3551	·3535	·3518	·3502	·3486	·3469	·3453	·3437	3	5	8	11	14
70°	·3420	·3404	·3387	·3371	·3355	·3338	·3322	·3305	·3289	·3272	3	5	8	11	14
71°	·3256	·3239	·3223	·3206	·3190	·3173	·3156	·3140	·3123	·3107	3	6	8	11	14
72°	·3090	·3074	·3057	·3040	·3024	·3007	·2990	·2974	·2957	·2940	3	6	8	11	14
73°	·2924	·2907	·2890	·2874	·2857	·2840	·2823	·2807	·2790	·2773	3	6	8	11	14
74°	·2756	·2740	·2723	·2706	·2689	·2672	·2656	·2639	·2622	·2605	3	6	8	11	14
75°	·2588	·2571	·2554	·2538	·2521	·2504	·2487	·2470	·2453	·2436	3	6	8	11	14
76°	·2419	·2402	·2385	·2368	·2351	·2334	·2317	·2300	·2284	·2267	3	6	8	11	14
77°	·2250	·2233	·2215	·2198	·2181	·2164	·2147	·2130	·2113	·2096	3	6	9	11	14
78°	·2079	·2062	·2045	·2028	·2011	·1994	·1977	·1959	·1942	·1925	3	6	9	11	14
79°	·1908	·1891	·1874	·1857	·1840	·1822	·1805	·1788	·1771	·1754	3	6	9	11	14
80°	·1736	·1719	·1702	·1685	·1668	·1650	·1633	·1616	·1599	·1582	3	6	9	11	14
81°	·1564	·1547	·1530	·1513	·1495	·1478	·1461	·1444	·1426	·1409	3	6	9	12	14
82°	·1392	·1374	·1357	·1340	·1323	·1305	·1288	·1271	·1253	·1236	3	6	9	12	14
83°	·1219	·1201	·1184	·1167	·1149	·1132	·1115	·1097	·1080	·1063	3	6	9	12	14
84°	·1045	·1028	·1011	·0993	·0976	·0958	·0941	·0924	·0906	·0889	3	6	9	12	14
85°	·0872	·0854	·0837	·0819	·0802	·0785	·0767	·0750	·0732	·0715	3	6	9	12	14
86°	·0698	·0680	·0663	·0645	·0628	·0610	·0593	·0576	·0558	·0541	3	6	9	12	15
87°	·0523	·0506	·0488	·0471	·0454	·0436	·0419	·0401	·0384	·0366	3	6	9	12	15
88°	·0349	·0332	·0314	·0297	·0279	·0262	·0244	·0227	·0209	·0192	3	6	9	12	15
89°	·0175	·0157	·0140	·0122	·0105	·0087	·0070	·0052	·0035	·0017	3	6	9	12	15

NATURAL TANGENTS.

Angle.	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
0°	0·0000	·0017	·0035	·0052	·0070	·0087	·0105	·0122	·0140	·0157	3	6	9	12	15
1°	0·0175	·0192	·0209	·0227	·0244	·0262	·0279	·0297	·0314	·0332	3	6	9	12	15
2°	0·0349	·0367	·0384	·0402	·0419	·0437	·0454	·0472	·0489	·0507	3	6	9	12	15
3°	0·0524	·0542	·0559	·0577	·0594	·0612	·0629	·0647	·0664	·0682	3	6	9	12	15
4°	0·0699	·0717	·0734	·0752	·0769	·0787	·0805	·0822	·0840	·0857	3	6	9	12	15
5°	0·0875	·0892	·0910	·0928	·0945	·0963	·0981	·0998	·1016	·1033	3	6	9	12	15
6°	0·1051	·1069	·1086	·1104	·1122	·1139	·1157	·1175	·1192	·1210	3	6	9	12	15
7°	0·1228	·1246	·1263	·1281	·1299	·1317	·1334	·1352	·1370	·1388	3	6	9	12	15
8°	0·1405	·1423	·1441	·1459	·1477	·1495	·1512	·1530	·1548	·1566	3	6	9	12	15
9°	0·1584	·1602	·1620	·1638	·1655	·1673	·1691	·1709	·1727	·1745	3	6	9	12	15
10°	0·1763	·1781	·1799	·1817	·1835	·1853	·1871	·1890	·1908	·1926	3	6	9	12	15
11°	0·1944	·1962	·1980	·1998	·2016	·2035	·2053	·2071	·2089	·2107	3	6	9	12	15
12°	0·2126	·2144	·2162	·2180	·2199	·2217	·2235	·2254	·2272	·2290	3	6	9	12	15
13°	0·2309	·2327	·2345	·2364	·2382	·2401	·2419	·2438	·2456	·2475	3	6	9	12	15
14°	0·2493	·2512	·2530	·2549	·2568	·2586	·2605	·2623	·2642	·2661	3	6	9	12	16
15°	0·2679	·2698	·2717	·2736	·2754	·2773	·2792	·2811	·2830	·2849	3	6	9	13	16
16°	0·2867	·2886	·2905	·2924	·2943	·2962	·2981	·3000	·3019	·3038	3	6	9	13	16
17°	0·3057	·3076	·3096	·3115	·3134	·3153	·3172	·3191	·3211	·3230	3	6	10	13	16
18°	0·3249	·3269	·3288	·3307	·3327	·3346	·3365	·3385	·3404	·3424	3	6	10	13	16
19°	0·3443	·3463	·3482	·3502	·3522	·3541	·3561	·3581	·3600	·3620	3	7	10	13	16
20°	0·3640	·3659	·3679	·3699	·3719	·3739	·3759	·3779	·3799	·3819	3	7	10	13	17
21°	0·3839	·3859	·3879	·3899	·3919	·3939	·3959	·3979	·4000	·4020	3	7	10	13	17
22°	0·4040	·4061	·4081	·4101	·4122	·4142	·4163	·4183	·4204	·4224	3	7	10	14	17
23°	0·4245	·4265	·4286	·4307	·4327	·4348	·4369	·4390	·4411	·4431	3	7	10	14	17
24°	0·4452	·4473	·4494	·4515	·4536	·4557	·4578	·4599	·4621	·4642	4	7	11	14	18
25°	0·4663	·4684	·4706	·4727	·4748	·4770	·4791	·4813	·4834	·4856	4	7	11	14	18
26°	0·4877	·4899	·4921	·4942	·4964	·4986	·5008	·5029	·5051	·5073	4	7	11	15	18
27°	0·5095	·5117	·5139	·5161	·5184	·5206	·5228	·5250	·5272	·5295	4	7	11	15	18
28°	0·5317	·5340	·5362	·5384	·5407	·5430	·5452	·5475	·5498	·5520	4	8	11	15	19
29°	0·5543	·5566	·5589	·5612	·5635	·5658	·5681	·5704	·5727	·5750	4	8	12	15	19
30°	0·5774	·5797	·5820	·5844	·5867	·5890	·5914	·5938	·5961	·5985	4	8	12	16	20
31°	0·6009	·6032	·6056	·6080	·6104	·6128	·6152	·6176	·6200	·6224	4	8	12	16	20
32°	0·6249	·6273	·6297	·6322	·6346	·6371	·6395	·6420	·6445	·6469	4	8	12	16	20
33°	0·6494	·6519	·6544	·6569	·6594	·6619	·6644	·6669	·6694	·6720	4	8	13	17	21
34°	0·6745	·6771	·6796	·6822	·6847	·6873	·6899	·6924	·6950	·6976	4	9	13	17	21
35°	0·7002	·7028	·7054	·7080	·7107	·7133	·7159	·7186	·7212	·7239	4	9	13	18	22
36°	0·7265	·7292	·7319	·7346	·7373	·7400	·7427	·7454	·7481	·7508	5	9	14	18	23
37°	0·7536	·7563	·7590	·7618	·7646	·7673	·7701	·7729	·7757	·7785	5	9	14	18	23
38°	0·7813	·7841	·7869	·7898	·7926	·7954	·7983	·8012	·8040	·8069	5	9	14	19	24
39°	0·8098	·8127	·8156	·8185	·8214	·8243	·8273	·8302	·8332	·8361	5	10	15	20	24
40°	0·8391	·8421	·8451	·8481	·8511	·8541	·8571	·8601	·8632	·8662	5	10	15	20	25
41°	0·8693	·8724	·8754	·8785	·8816	·8847	·8878	·8910	·8941	·8972	5	10	16	21	26
42°	0·9004	·9036	·9067	·9099	·9131	·9163	·9195	·9228	·9260	·9293	5	11	16	21	27
43°	0·9325	·9358	·9391	·9424	·9457	·9490	·9523	·9556	·9590	·9623	6	11	17	22	28
44°	0·9657	·9691	·9725	·9759	·9793	·9827	·9861	·9896	·9930	·9965	6	11	17	23	29

NATURAL TANGENTS.

Angle	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
45°	1·0000	1·0035	1·0070	1·0105	1·0141	1·0176	1·0212	1·0247	1·0283	1·0319	6	12	18	24	30
46°	1·0355	1·0392	1·0428	1·0464	1·0501	1·0538	1·0575	1·0612	1·0649	1·0686	6	12	18	25	31
47°	1·0724	1·0761	1·0799	1·0837	1·0875	1·0913	1·0951	1·0990	1·1028	1·1067	6	13	19	25	32
48°	1·1106	1·1145	1·1184	1·1224	1·1263	1·1303	1·1343	1·1383	1·1423	1·1463	7	13	20	26	33
49°	1·1504	1·1544	1·1585	1·1626	1·1667	1·1708	1·1750	1·1792	1·1833	1·1875	7	14	21	28	34
50°	1·1918	1·1960	1·2002	1·2045	1·2088	1·2131	1·2174	1·2218	1·2261	1·2305	7	14	22	29	36
51°	1·2349	1·2393	1·2437	1·2482	1·2527	1·2572	1·2617	1·2662	1·2708	1·2753	8	15	23	30	38
52°	1·2799	1·2846	1·2892	1·2938	1·2985	1·3032	1·3079	1·3127	1·3175	1·3222	8	16	24	31	39
53°	1·3270	1·3319	1·3367	1·3416	1·3465	1·3514	1·3564	1·3613	1·3663	1·3713	8	16	25	33	41
54°	1·3764	1·3814	1·3865	1·3916	1·3968	1·4019	1·4071	1·4124	1·4176	1·4229	9	17	26	34	43
55°	1·4281	1·4335	1·4388	1·4442	1·4496	1·4550	1·4605	1·4659	1·4715	1·4770	9	18	27	36	45
56°	1·4826	1·4882	1·4938	1·4994	1·5051	1·5108	1·5166	1·5224	1·5282	1·5340	10	19	29	38	48
57°	1·5399	1·5458	1·5517	1·5577	1·5637	1·5697	1·5757	1·5818	1·5880	1·5941	10	20	30	40	50
58°	1·6003	1·6066	1·6128	1·6191	1·6255	1·6319	1·6383	1·6447	1·6512	1·6577	11	21	32	43	53
59°	1·6643	1·6709	1·6775	1·6842	1·6909	1·6977	1·7045	1·7113	1·7182	1·7251	11	23	34	45	56
60°	1·7321	1·7391	1·7461	1·7532	1·7603	1·7675	1·7747	1·7820	1·7893	1·7966	12	24	36	48	60
61°	1·8040	1·8115	1·8190	1·8265	1·8341	1·8418	1·8495	1·8572	1·8650	1·8728	13	26	38	51	64
62°	1·8807	1·8887	1·8967	1·9047	1·9128	1·9210	1·9292	1·9375	1·9458	1·9542	14	27	41	55	68
63°	1·9626	1·9711	1·9797	1·9883	1·9970	2·0057	2·0145	2·0233	2·0323	2·0413	15	29	44	58	73
64°	2·0503	2·0594	2·0686	2·0778	2·0872	2·0965	2·1060	2·1155	2·1251	2·1348	16	31	47	63	78
65°	2·1445	2·1543	2·1642	2·1742	2·1842	2·1943	2·2045	2·2148	2·2251	2·2355	17	34	51	68	85
66°	2·2460	2·2566	2·2673	2·2781	2·2889	2·2998	2·3109	2·3220	2·3332	2·3445	18	37	55	73	92
67°	2·3559	2·3673	2·3789	2·3906	2·4023	2·4142	2·4262	2·4383	2·4504	2·4627	20	40	60	79	99
68°	2·4751	2·4876	2·5002	2·5129	2·5257	2·5386	2·5517	2·5649	2·5782	2·5916	22	43	65	87	108
69°	2·6051	2·6187	2·6325	2·6464	2·6605	2·6746	2·6889	2·7034	2·7179	2·7326	24	47	71	95	119
70°	2·7475	2·7625	2·7776	2·7929	2·8083	2·8239	2·8397	2·8556	2·8716	2·8878	26	52	78	104	130
71°	2·9042	2·9208	2·9375	2·9544	2·9714	2·9887	3·0061	3·0237	3·0415	3·0595	29	58	87	116	144
72°	3·0777	3·0961	3·1146	3·1334	3·1524	3·1716	3·1910	3·2106	3·2305	3·2506	32	64	96	129	161
73°	3·2709	3·2914	3·3122	3·3332	3·3544	3·3759	3·3977	3·4197	3·4420	3·4646	36	72	108	144	180
74°	3·4874	3·5105	3·5339	3·5576	3·5816	3·6059	3·6305	3·6554	3·6806	3·7062	41	81	122	163	204
75°	3·7321	3·7583	3·7848	3·8118	3·8391	3·8667	3·8947	3·9232	3·9520	3·9812					
76°	4·0108	4·0408	4·0713	4·1022	4·1335	4·1653	4·1976	4·2303	4·2635	4·2972					
77°	4·3315	4·3662	4·4015	4·4374	4·4737	4·5107	4·5483	4·5864	4·6252	4·6646					
78°	4·7046	4·7453	4·7867	4·8288	4·8716	4·9152	4·9594	5·0045	5·0504	5·0970					
79°	5·1446	5·1929	5·2422	5·2924	5·3435	5·3955	5·4486	5·5026	5·5578	5·6140					
80°	5·6713	5·7297	5·7894	5·8502	5·9124	5·9758	6·0405	6·1066	6·1742	6·2432	**Mean differences not sufficiently accurate.**				
81°	6·3138	6·3859	6·4596	6·5350	6·6122	6·6912	6·7720	6·8548	6·9395	7·0264					
82°	7·1154	7·2066	7·3002	7·3962	7·4947	7·5958	7·6996	7·8062	7·9158	8·0285					
83°	8·1443	8·2636	8·3863	8·5126	8·6427	8·7769	8·9152	9·0579	9·2052	9·3572					
84°	9·5144	9·6768	9·8448	10·019	10·199	10·385	10·579	10·780	10·988	11·205					
85°	11·430	11·664	11·909	12·163	12·429	12·706	12·996	13·300	13·617	13·951					
86°	14·301	14·669	15·056	15·464	15·895	16·350	16·832	17·343	17·886	18·464					
87°	19·081	19·740	20·446	21·205	22·022	22·904	23·859	24·898	26·031	27·271					
88°	28·636	30·145	31·821	33·694	35·801	38·188	40·917	44·066	47·740	52·081					
89°	57·290	63·657	71·615	81·847	95·489	114·59	143·24	190·98	286·48	572·96					

The following table was supplied by Messrs. Crofts :

CONVERSION TABLES FOR DIAMETRAL AND CIRCULAR PITCHES OF TEETH FOR GEAR WHEELS

Circular Pitch.	*Diametral Pitch.*	*Diametral Pitch.*	*Circular Pitch.*
in.			in.
4	·7854	20	·1571
3½	·8976	19	·1653
3	1·0472	18	·1745
2¾	1·1424	17	·1848
2½	1·2566	16	·1963
2¼	1·3963	15	·2094
2	1·5708	14	·2244
1⅞	1·6755	13	·2416
1¾	1·7952	12	·2618
1⅝	1·9333	11	·2856
1½	2·0944	10	·3142
1⅜	2·2848	9	·3491
1¼	2·5133	8	·3927
1⅛	2·7925	7	·4488
1	3·1416	6	·5236
15/16	3·3510	5	·6283
⅞	3·5904	4	·7854
13/16	3·8666	3½	·8976
¾	4·1888	3	1·0472
11/16	4·5696	2¾	1·1424
⅝	5·0265	2½	1·2566
9/16	5·5851	2¼	1·3963
½	6·2832	2	1·5708
7/16	6·1808	1¾	1·7952
⅜	8·3776	1½	2·0944
5/16	10·0531	1¼	2·5133
¼	12·5664	1	3·1416

Formulæ for M/Cut Gear Wheel Teeth

Diametral pitch $= \dfrac{3{\cdot}1416}{\text{circular pitch}}$.

Circular pitch $= \dfrac{3{\cdot}1416}{\text{diam. pitch}}$.

Pitch diameter $= \dfrac{\text{no. of teeth}}{\text{diam. pitch}}$ or $\dfrac{\text{no. of teeth} \times \text{cir. pitch}}{3{\cdot}1416}$.

No. of teeth $=$ pitch diam. × diam. pitch ; or $\dfrac{\text{pitch diam.} \times 3{\cdot}1416}{\text{circular pitch}}$.

Thickness of tooth at pitch line $= \dfrac{\text{circular pitch}}{2}$ or $\dfrac{1{\cdot}57}{\text{diam. pitch}}$.

Height of tooth above pitch line, or addendum $= {\cdot}3183$ cir. pitch or $\dfrac{1}{\text{diam. pitch}}$.

Working depth of tooth $= {\cdot}6366$ cir. pitch or $\dfrac{2}{\text{diam. pitch}}$.

Whole depth of tooth $= {\cdot}6866$ cir. pitch or $\dfrac{2{\cdot}157}{\text{diam. pitch}}$.

Clearance at bottom of tooth = one-tenth of thickness of tooth at pitch line.

Distance between the centres of a pair of wheels

$$= \frac{\text{sum of number of teeth in both wheels}}{2 \times \text{diam. pitch}}$$

$$\text{or } \frac{\text{sum of pitch diameters of both wheels}}{2}.$$

Width of face $= \dfrac{8}{\text{diam. pitch}}$ to $\dfrac{10}{\text{diam. pitch}}$ or 2½ to 3½ cir. pitch.

Conversion Tables (English-Metric)

Metric to English.		*English to Metric.*	
Millimetres to Inches	÷ 25·4	Inches to Millimetres	× 25·4
Centimetres to Inches	÷ 2·54	Inches to Centimetres	× 2·54
Metres to Inches	× 39·37	Inches to Metres	÷ 39·37
Metres to Feet	× 3·281	Feet to Metres	÷ 3·281
Kilometres to Miles	× ·621	Miles to Kilometres	÷ ·621
Kilometres to Feet	× 3280·9	Feet to Kilometres	÷ 3280·9
Sq. Millimetres to Sq. Inches	÷ 645·16	Sq. Inches to Sq. Millimetres	× 645·16
Sq. Centimetres to Sq. Inches	÷ 6·45	Sq. Inches to Sq. Centimetres	× 6·45
Cubic Centimetres to Cubic Inches	× ·06	Cubic Inches to Cubic Centimetres	× 16·39
Kilogrammes to Pounds	× 2·2046	Pounds to Kilogrammes	× ·454
Kilogrammes to Tons	× ·001	Tons to Kilogrammes	÷ ·001
Kilogrammes per Sq. Centimetre to Lb. per Sq. Inch	× 14·223	Pounds per Sq. Inch to Kilogrammes per Sq. Centimetre	× ·0703
Kilogramme/Metres to Foot/Pounds	× 7·223	Foot/Pounds to Kilogramme/Metres	× ·1382
Kilogrammes per Cheval to Pounds per H.P.	× 2·235	Pounds per H.P. to Kilogrammes per Cheval	× ·447
Kilowatts to H.P.	× 1·34	H.P. to Kilowatts	÷ 1·34
Watts to H.P.	÷ 746	H.P. to Watts	× 746
Cheval Vapeur to H.P.	× ·9863	H.P. to Cheval Vapeur	× 1·014
Atmospheres to Lb. per Sq. Inch	× 14·7	Lb. per Sq. Inch to Atmospheres	÷ 14·7

÷ Divide

× Multiply

ANSWERS

Exercise (1)

(1) $\frac{3}{32}$. (2) $10\frac{3}{4}$. (3) $3\frac{3}{13}$. (4) $1\frac{5}{8}$.
(5) $\frac{31}{64}$. (6) $\frac{3}{32}$. (7) $\frac{25}{32}$. (8) $\frac{13}{64}$.
(9) $\frac{15}{64}$. (10) 1. (11) $4\frac{2}{3}$. (12) $\frac{6}{11}$.
(13) $\frac{5}{16}$. (14) $\frac{3}{64}$. (15) $2\frac{1}{2}$. (16) $3\frac{13}{20}$.
(17) $\frac{3}{32}$. (18) $3\frac{1}{8}$. (19) $3\frac{3}{4}$. (20) $\frac{29}{32}$.
(21) $86\frac{1}{16}''$. (22) 26 lb. 9 oz. (23) 54". (24) $3\frac{3}{8}'' \times 3\frac{3}{8}''$.
(25) $\frac{7}{8}$. (26) $23\frac{1}{3}$, $11\frac{2}{3}$, $9\frac{1}{3}$. (27) $18\frac{2}{3}$. (28) $\frac{14}{297}$.
(29) $\frac{52}{81}$. (30) $1\frac{1}{9}$. (31) $89\frac{7}{12}''$. (32) $3\frac{33}{64}$.

Exercise (2)

(1) (*a*) 720·3. (*b*) 2·73. (*c*) 200. (*d*) 790.
(2) (*a*) 2·7394. (*b*) 0·7947. (*c*) 0·00565. (*d*) 0·0003472.
(3) (*a*) 22·5. (*b*) 0·09875. (*c*) 1·68. (*d*) 75. (*e*) 0·075. (*f*) 750,000
(4) (*a*) 18·75. (*b*) 0·225. (*c*) 12·5.
(5) (*a*) 48. (*b*) ·175. (*c*) 1·6325.

(6)

Core diameter, inch	·186	0·622	0·3932	0·84	1·2866

(7) (*a*) 16. (*b*) 8. (*c*) 6. (*d*) 4.
(8) (*a*) 1·434". (*b*) 9·22". (*c*) 14·37".
(9) 114·3 mm. bore, 188 mm. long.
(11) 0·703. (12) 1·52. (13) 56·42.
(14) 0·0368. (15) 16. (16) 3·542 lb.
(17) 16,464 lb. (18) 0·629. (19) 3·56.
(20) The first and the last are equal amounts. They exceed the middle one by 2*s*. 6*d*.
(21) 0·875.

Exercise (3)

(1) 3·405. (2) £10 8*s*. 4*d*. (3) 6 stones 4 lb. 13 oz
(4) $7\frac{3}{4}$ tons. (5) 18*s*. $2\frac{3}{8}$*d*. (6) 9 stones 0 lb. 15 oz
(7) 2*s*. 11*d*. (8) 66, 132, 264 yds. per min.
(9) (*a*) 3·13, 3·68, 4·42, 3·84. (*b*) 3·56.
(10) $247\frac{3}{8}$. (11) $218\frac{1}{4}$°. (12) 80.
(13) 7·656. (14) 0·2314. (15) £42 1*s*. $10\frac{1}{2}$*d*.
(16) 0·151". (17) 148·6 hrs. (18) 17*s*. $4\frac{1}{2}$*d*.

Exercise (4)

(1) $4x + 12 = 36$. $x = 6$. (2) $108 - 9x = 18$. $x = 10$.
(3) $\frac{x}{11} + 51 = 203$. $x = 1672$. (4) $5 + 1\frac{1}{2}x = 17$. $x = 8$.
(5) $\frac{22}{7}D = 11$. $D = 3\frac{1}{2}''$.

Exercise (5)

(1) 625. (2) 169. (3) 4·41. (4) 1·44.
(5) 27·7729. (6) 0·6084. (7) 100·2001. (8) 4199·04.
(9) 8. (10) 9·07. (11) 1·99. (12) 0·401.
(13) $\frac{7}{8}$. (14) $\frac{3}{2}$. (15) $3\frac{1}{2}$. (16) $1\frac{3}{8}$.

Exercise (6)

(1) 8. (2) 76. (3) 40 seconds. (4) 54.
(5) 80. (6) 80. (7) 100. (8) $3\frac{3}{4}''$.
(9) $96\frac{1}{4}$. (10) 27″. (11) 30 cwt. copper, 15 cwt. tin.
(12) $267\frac{3}{4}$ cu. ft. (13) x £187, y £110, z £55.
(14) Copper 94·08 lb., tin 13·44 lb., antimony 4·48 lb.
(15) W = $5\frac{1}{3}$. (16) £3 3*s*.
(17) Steel 48·6 tons, copper 4·32 tons, other metals 1·08 tons.
(18) At a point $4\frac{3}{8}''$ from the fulcrum on the other side.
(19) On the same diameter, but 8″ on the other side of the centre.
(20) 0·6 and 0·8. (21) 992. (22) 10·25 should be 11·25.
(23) $33\frac{1}{3}$ tons of steel, $1\frac{2}{3}$ tons of chromium. (24) $\frac{3}{8}$.
(25) $1\frac{1}{3}$. (26) 20 yards. (27) £149 16*s*. 10*d*.
(28) At a point 15″ on the other side of the centre.

Exercise (7)

(1) 7·874. (2) 5·3. (3) 1·77″, 11·02″.
(4) 16. (5) 27·43. (6) 11·81, $20\frac{4}{5}$, 20, $28\frac{4}{5}$
(7) The 8″. (8) 15·902. (9) 45·45.
(10) 158·4 lb. (11) 31·42. (12) 2496.
(13) 39·98. (14) 1·27. (15) 7·8.
(16) 589·5. (17) 2993.
(18) $2{\cdot}116\begin{smallmatrix}+\,0{\cdot}0008\\-\,0{\cdot}0008\end{smallmatrix}$. Metric tolerance ·004 cm. Inch tolerance ·0016.

Exercise (8)

(1) 20·9. (2) 9. (3) 40.
(4) 0·38. (5) 76·2, 22·2. 1·6. (6) 59 tons 8 cwt.
(7) 28 lb. (8) 144 tons. (9) $142\frac{1}{2}$ tons.
(10) $28\frac{1}{8}$%, 64%. (11) 30%. (12) $1\frac{1}{4}$%, $3\frac{1}{3}$%, $2\frac{1}{12}$%.
(13) 41·4%. (14) 7·7. (15) 1·6, 22·2, and 76·2.
(16) 20 tons. (17) 7*s*. 6*d*. (18) 16%.
(19) 160 cu. in. and 24 cu. in respectively. (20) £85 14*s*. 9*d*.
(21) 2·16″. (22) 800.
(23) 84% iron, 12% chromium, 4% nickel.

Exercise (9)

(6) $165\frac{3}{8}$, $68\frac{1}{2}$, $46\frac{1}{2}$, $25\frac{1}{8}$. (7) 7,700, 18,000, 35,000, 25,000.
(8) $1\frac{1}{8}''$, $2\frac{1}{2}''$, 1·1424, 2·5133. (9) 8, 80.
(11) 14, 9, 7, 5, $3\frac{1}{2}$. (13) 3·2 pence, 1570.

Exercise (10) (a)

(1) 3·192″. (2) $18\frac{1}{3}$. (3) 26·7.
(4) 600,000. (5) 628·8, 2863. (6) 372·7.
(7) 20. (8) 23·3. (9) 32·5.
(10) 1·3. (11) 4 mins. 16 sec. (12) 1·856 cu. in.
(13) 2·645″. (14) 560. (15) 2*s*. 11*d*.
(16) 257,000. (17) $y = 11{\cdot}05$. (18) 97,920.
(19) 0·0058. (20) ·97 ton.
(21) (i) 212. (ii) −40. (iii) 1452. (iv) −183.
(22) 18·75 ft. per sec. (23) 40·3 lb.

Exercise (10) (b)

(1) $12x + 8y$. (2) $30m - 18n$.
(3) $3L^3 - 2L^2$. (4) $5b^5 + 3b^4 - 2b^3$.
(5) $12x^4y + 8x^2y^2 - 2xy^4$. (6) $9x^2 - 4y^2$.
(7) $x^3 + y^3$. (8) $x^3 - y^3$.
(9) $5a$. (10) $2m - n - P$.
(11) $a^2b + 7a - 4$. (12) $3a + 2P + 4$.
(13) $3a^2b + 3s - 1$. (14) $- 5m^2n^2 + 2mn - 4$.
(15) $2C^2$. (16) $4x^2 - 3x$. (17) $30N - 15$.
(18) $8d^2 - d$. (19) $- 40 + P^2$. (20) $- 4 + 5R^2$.
(21) $- 2m + 8n$. (22) $a^2 + ab + x^2 - xy$.
(23) $7x - 5y$. (24) $7a - 7$. (25) $7m + 7$.

Exercise (10) (c)

(1) $y = 8$. (2) $y = - 4$. (3) $v = 11$. (4) $M = 36$. (5) $L = 7.37$.

Exercise (10) (d)

(1) 7. (2) 3. (3) 5. (4) 4.

Exercise (10) (e)

(1) 2. (2) 15. (3) $\frac{1}{4}$. (4) $\frac{2}{3}$.
(5) $\frac{1}{6}$. (6) (i) $37\frac{1}{2}$. (ii) 60. (iii) $\frac{2}{15}$.
(7) 340. (8) (i) 299·4 sq. in. (ii) 18·71 sq. in.
(9) (i) 4. (ii) 1. (iii) $- \frac{13}{24}$ or $- \cdot 541$. (10) 4. (11) $6\frac{1}{25}$.

Exercise (10) (f)

(1) $b = 36$. (2) $y = 37\frac{1}{2}$. (3) $z = 60$. (4) $m = 3\frac{1}{3}$.
(5) (*a*) $\pi x N$. (*b*) $\frac{\pi x N}{60}$. (*c*) $\frac{60 . \pi x N}{5280}$. (6) $a = 12$.
(7) $r = 7$. (8) $L = 30''$, $B = 27''$. (9) 4 seconds.
(10) $\frac{1}{2}$ second. (11) $r = 3\frac{1}{2}$. (12) $C = 10$. $- 40$.
(13) 3600. (14) $y = - \frac{28}{48}$. (15) Lathe £200, drilling machine £50.
(16) $t = 140$. (17) 80 and 160. (18) 10″, 12″, 15″. (19) 3″.
(20) 6″ and 4″. (21) $L - \frac{n(6 + \frac{1}{8})}{12}$. 7′ 11″.
(22) (*a*) $M = 9$. (*b*) $a = 13$. (23) $N = 7$, $R = 1 \cdot 996$.
(24) $L = 10$, $V = - 2$. (25) $F = 0 \cdot 012W + 3$. 17·4 lb.
(26) $\frac{16}{25}$ ton, $\frac{9}{25}$ ton. (27) (*a*) 16° 28′. (*b*) 2·4772.

Exercise (11)

(2) 360°.
(9) (36° 48′). (53° 12′). 90°.
(10) 0·866.
(11) 4·74″.
(12) 4·88″.
(13) $4\frac{1}{2}$″.
(15) 120°.
(16) Angle across 40°. Other angles 140°.
(17) 3·56″, 8·32″.
(18) $75\frac{1}{2}$°, $105\frac{1}{2}$°.

Exercise (19)

(5) 1680. (6) 3610 B.T.U. (7) 1552 B.T.U.
(8) 2·9 per cent. approx. (10) 55,000 ft./lb., 70·7 B.T.U.
(11) 13,440 ft./lb., 17·3 B.T.U. (23) 0·109.
(24) 0·0879. (25) (*a*) 15,360,000. (*b*) 15,360 lb.

Exercise (20)

(4) 450. (5) 1482. (8) 0·603 inch.
(9) 0·14454″. (10) (*b*) 0·219″. (12) (*b*) 0·08184″ increase.
(13) 433°. (15) (*c*) 1085°.

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